1 Problems by hand from Chapter 4

Do problems 4.3, 4.6, and 4.8 on page 80 in Chapra (2005) by hand. Follow the steps outlined in the course syllabus.

2 The Physics of Golf

When a golfer hits a golf ball, the ball launches with a certain initial velocity, angle of attack, and spin. As the ball flies through the air, gravity pulls the ball down, drag slows the ball down, and spin creates a lift force that either lifts the ball, drops the ball or curves the ball. Figure 1 shows the golf ball problem with a free-body diagram of the ball at an intermediate point in the flight path.

From Newton’s first law, we know that

$$\ddot{a} = \frac{d\vec{v}}{dt} = \frac{\vec{F}}{m}$$ (1)

To track the 2-D trajectory of a golf ball (i.e. no hook or slice), we must keep track of its $x$ and $y$ position and, therefore, also its $x$-velocity component $v_x$ and $y$-velocity component $v_y$. Let $\alpha$ be the instantaneous angle of attack. Applying Newton’s law, the governing equations
Figure 1: Diagram of the flight of a golf ball and the acting forces.

The equations of motion for the golf ball become

\[
\frac{dv_x}{dt} = -\frac{F_L}{m} \sin \alpha - \frac{F_D}{m} \cos \alpha \\
\frac{dv_y}{dt} = \frac{F_L}{m} \cos \alpha - \frac{F_D}{m} \sin \alpha - g \\
\frac{dx}{dt} = v_x \\
\frac{dy}{dt} = v_y
\]

(2)

where \(F_L\) is a lift force and \(F_D\) is a drag force; \(g\) is the acceleration of gravity, and \(m\) is the mass of the golf ball (0.0455 kg).

The drag and lift laws for golf balls are given by the formulas

\[
F_D = \frac{1}{2} \rho v^2 \left( \frac{\pi d^2}{4} \right) C_D
\]

(3)

\[
F_L = \frac{1}{2} \rho v^2 \left( \frac{\pi d^2}{4} \right) C_L
\]

(4)

where \(\rho\) is the density of air (1.227 kg/m\(^3\)), \(v\) is the total velocity \((v = \sqrt{v_x^2 + v_y^2})\), and \(d\) is the diameter of the golf ball (4.27 cm). \(C_D\) and \(C_L\) are the drag and lift coefficients. For dimpled golf balls, the drag coefficient is between 0.2 and 0.5, and the magnitude of the lift coefficient is between 0.0 and 0.4, depending on the spin on the ball. For top-spin, the lift coefficient is negative and for backspin the lift coefficient is positive.

The initial conditions for the ball result from a collision between the club head and the golf ball. This gives the ball an initial velocity, angle of attack and spin. To compute the initial velocity, we conserve momentum during the collision. However, because both the club head and golf ball deform during the collision, energy is lost and the collision is not perfectly elastic. It turns out that the energy loss for a collision between two materials is linearly related to the velocity of the two objects. For a golf ball and golf club, this is summarized...
as the coefficient of restitution, given by

\[ e_{COR} = \frac{v_{1,b} - v_{1,c}}{v_{0,c}} \]  

(5)

where \( v_{1,c} \) is the velocity of the club head after striking the ball, \( v_{0,c} \) is the club head speed before collision, and \( v_{1,b} \) is the launch-velocity of the ball. Because this equation has two unknowns (\( v_{1,b} \) and \( v_{1,c} \)), it must be solved together with the conservation of momentum, given by

\[ m_b v_{1,b} + m_c v_{1,c} = m_c v_{0,c} \]  

(6)

where \( m_b \) and \( m_c \) are the mass of the golf ball and club, respectively. The initial angle of attack is the launch angle reported on each club, and spin depends on the skill of the golfer. Golfers with higher club-head speed generate greater spin.

Modify the bungee jumper programs (Download from Calendar web page) to compute and display the trajectory of the golf ball given these equations. Here are a few items to keep in mind:

- The variables appearing in the differential equations that have to be passed to the Matlab function that computes the right-hand-side (the modified \texttt{bungee.m} file) are the drag and lift coefficient, the diameter and mass of the golf ball, and the density of the air. The remaining design variables (the club-head speed, mass of the club-head, coefficient of restitution, and launch angle) are only used to determine the initial condition for \( v_x \) and \( v_y \)—they do not need to be passed to the differential equation solver.

- The numerical method variables are: the initial velocities \( v_{x,0} \) and \( v_{y,0} \), the initial positions \( x_0 \) and \( y_0 \), the initial time \( t_0 \), the step size \( \Delta t \), and the final time to calculate \( t_f \).

- You will need to rename the function \texttt{diffeq2.m} to \texttt{diffeq4.m} and modify the file so that it computes the solution for a coupled set of four differential equations (e.g. Equation (2)).

- You will need to rename the function \texttt{bungee.m} and insert the equations for the golf ball problem. Note that \( \alpha \) does not equal \( \alpha_0 \) throughout the flight, but rather can be computed from the component velocity as \( \alpha = \tan^{-1}(v_y/v_x) \).

- You will need to modify the program \texttt{main.m} to manage the golfball problem inputs and outputs.

- The simulation should end when the golf ball hits the ground.

Once you have tested and de-bugged your program, run it for the following two cases:
1. Run your program to compare Tiger Woods’ drive shot to that of a weekend golfer. The main difference between the two golfers is their club-head speed. Because weekend golfers have a slower club-head speed, they generate less spin and, therefore, have a lower lift coefficient. Use the following data:

<table>
<thead>
<tr>
<th>Club</th>
<th>Tiger Woods</th>
<th>Amature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Club head speed</td>
<td>52 m/s</td>
<td>36 m/s</td>
</tr>
<tr>
<td>Club head weight</td>
<td>193 grams</td>
<td>198 grams</td>
</tr>
<tr>
<td>( e_{COR} )</td>
<td>0.830</td>
<td>0.800</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>Lift coefficient</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>Launch angle</td>
<td>11°</td>
<td>13°</td>
</tr>
</tbody>
</table>

Plot both trajectories on a single graph using `legend` to distinguish between the two flight paths.

2. To hit the ball varying distances, golfers use clubs with a different launch angle and shaft length. Ideally, the golfer should always make a full swing, but the shorter shafts result in slower club-head speeds and the higher lofts result in greater initial launch angle. Use the data in Table 1 to create a single plot with the trajectories for each club. You can annotate your figure using the buttons in the Matlab figure window. Figure 2 gives an overview of how to do this.

<table>
<thead>
<tr>
<th>Club</th>
<th>Club-head speed [m/s]</th>
<th>Club-head weight [g]</th>
<th>( e_{COR} )</th>
<th>Drag Coefficient [-]</th>
<th>Lift Coefficient [-]</th>
<th>Launch Angle [degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Wood</td>
<td>36.0</td>
<td>198</td>
<td>0.80</td>
<td>0.23</td>
<td>0.10</td>
<td>13</td>
</tr>
<tr>
<td>3 Wood</td>
<td>33.9</td>
<td>210</td>
<td>0.77</td>
<td>0.23</td>
<td>0.11</td>
<td>15</td>
</tr>
<tr>
<td>5 Wood</td>
<td>33.0</td>
<td>220</td>
<td>0.72</td>
<td>0.24</td>
<td>0.11</td>
<td>18</td>
</tr>
<tr>
<td>3 Iron</td>
<td>30.9</td>
<td>249</td>
<td>0.70</td>
<td>0.24</td>
<td>0.10</td>
<td>21</td>
</tr>
<tr>
<td>5 Iron</td>
<td>30.1</td>
<td>263</td>
<td>0.60</td>
<td>0.26</td>
<td>0.11</td>
<td>27</td>
</tr>
<tr>
<td>7 Iron</td>
<td>29.3</td>
<td>277</td>
<td>0.50</td>
<td>0.26</td>
<td>0.11</td>
<td>34</td>
</tr>
<tr>
<td>9 Iron</td>
<td>28.5</td>
<td>291</td>
<td>0.40</td>
<td>0.28</td>
<td>0.13</td>
<td>42</td>
</tr>
<tr>
<td>Sand Wedge</td>
<td>28.5</td>
<td>306</td>
<td>0.30</td>
<td>0.30</td>
<td>0.14</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 1: Golf club statistics for a typical weekend golfer.
Figure 2: Overview of editing function in the Matlab figure window.