CVEN 302
Computer Applications in Engineering and Construction
Programming Assignment #9
Eigenvalue analysis of a Shear Building Model

Date distributed : 12.4.2009
Date due : 12.7.2009 at 10:20 a.m.

Return your solution either in class or in my mail box (8th Floor, CE/TTI) by the date shown above. Please show all your work and follow the rules outlined in the course syllabus.

Every structure has a natural frequency—the frequency that leads to resonance. In the previous assignment, we analyzed the motion of a shear building model under earthquake loads. In this assignment, we will apply modal analysis and Eigenvalues to determine the natural frequency of the building motion and to identify the characteristic shapes for the building motions that can be expected for each Eigenfrequency. In the following section, we introduce the theory of modal analysis and pose the Eigenvalue problem. In Section 2, the details of the programming assignment are outlined. Section 3 presents the requirements of the written assignment, followed by Section 4, the challenge problem.

1 Modal Analysis

If we neglect damping and assuming all the building coefficients are constant, then the analytical solution to the building motion problem can be written as the series solution

\[ x = \sum_{i=1}^{m} a_i \cos(\omega_i t + \varphi_i) \phi_i \]  

where \( x \) is the displacement at each floor and \( \phi_i \) is a characteristic building shape for mode \( i \). Each mode of the building motion contributes to the total displacement \( x \) with an amplitude \( a_i \), frequency \( \omega_i \), and phase angle \( \varphi_i \). Hence, the complicated building motion can be simplified as the linear superposition of the building response to each characteristic frequency \( \omega_i \). For the numerical solution, we take \( m \) equal to the number of floors.

In order to solve for the natural frequencies \( \omega_i \) and shape functions \( \phi \), we consider the governing equation for the building motion:

\[ M \ddot{x} + Kx = 0. \]  

where \( M \) and \( K \) are the mass and stiffness matrices, respectively. When the right-hand-side of the governing equation is zero, there is no forcing, and we call the motion a free vibration. Substituting (1) into (2) gives

\[ -\omega_i^2 M \phi_i + K \phi_i = 0 \]  

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And rearranging in the form of an Eigenvalue problem gives
\[
\begin{bmatrix}
M^{-1}K - \omega_i^2 I
\end{bmatrix} \phi_i = 0
\] (4)
Thus, \( \omega_i^2 \) is the eigenvalue of the matrix \( A = M^{-1}K \).

2 Programming Assignment

For the shear building in the previous assignment, use Matlab to do the following:

1. Use the built-in function \texttt{eig} to compute the Eigenvalues and Eigenvectors of \( A = M^{-1}K \).

2. Plot the modal shapes \( \phi_i \) using the Eigenvectors returned by the Matlab \texttt{eig} function. Plot \( \phi_i \) on the \( x \)-axis and the floor number on the \( y \)-axis. These shapes are the characteristic shapes in which the building can move. Plot all 10 shape functions \( \phi_i \) in a single figure using \texttt{subplot}.

3. Report the values of the Eigenfrequencies \( \omega_i \) in their proper units. Are these values consistent with rapid or slow oscillations? Do you think this building is likely to experience a forcing in this range of frequencies during its design life? From what mechanism? What would likely be the result of that forcing?

3 Memorandum

For the text of the memorandum, include the plot of the mode shapes \( \phi_i \) and report the values of the Eigenfrequencies. Also structure the memorandum to answer each of the questions posed above. Include your program code in an appendix with appropriate text introducing each subprogram. Turn in a copy of the memorandum and email your program to the TA as a single program file.

4 Challenge

For the smallest eigenvalue, simulate the motion of the building using the code to your previous assignment and setting the damping coefficients to zero. That is, simulate the building motion with the ground acceleration equal to
\[
a_g = \sin(\omega_1 t)
\] (5)
Does the building shape match the Eigenvector shape \( \phi_1 \)? Do you see characteristics of resonance? Repeat this same exercise with the damping coefficients returned to their original values. Does the building continue to show signs of resonance? Why or why not?