Force on a Plane Surface II

Learning Objectives:
- Use the pressure prism method to find hydrostatic forces on submerged plane surfaces

Motivational Question:
- The space shuttle has a window. What is the net hydrostatic force on the window when in space?
CVEN 311 Fluid Dynamics
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Post Objectives to Projector.
Bring Calculator

Example: 2.51

1. y-axis as shown.
2. \( x_c = 0 \) (symmetric)
   \( y_c = 9' + \frac{6'}{2} = 12' \)
   \( h_c = y_c = 12' \) \((\theta = 90^\circ)\)
3. \( F_x = \rho h_c A \)
   \[ = 62.4 \frac{\text{lb}}{\text{ft}^2} \cdot 12 \text{ft} \cdot \left(\pi \left(\frac{\frac{6}{2} \text{ft}}{2}\right)^2\right) \]
   \[ = 21,171.8 \text{ lb} \]
Example, cont.:

\[ I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (3)^4}{4} \text{ ft}^4 \]
\[ = 63.62 \text{ ft}^4 \]
\[ I_{xc} = 0 \quad \text{(symmetric)} \]
\[ y_c = \frac{I_{xc}}{y_c A} + y_e \]
\[ = \frac{63.62 \text{ ft}^4}{(12 \text{ ft})(\pi (3 \text{ ft})^2)} + 12 \text{ ft} \]
\[ = 12.1875 \text{ ft} \]

Pressure Prism:

If the plane of interest is uniform into the page, then simple graphical method can be used:

[Diagram of hydrostatic pressure distribution with labeled variables]
A large, open tank contains water and is connected to a 6-ft diameter conduit as shown in Fig. P2.51. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.

\[
F_R = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right) (12 \text{ ft})(\frac{\pi}{4})(6 \text{ ft})^2 = 21,200 \text{ lb}
\]

\[
y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad I_{xc} = \frac{\pi}{4} (3 \text{ ft})^4 = 63.6 \text{ ft}^4
\]

Thus,

\[
y_R = \frac{\pi}{4} (3 \text{ ft})^4 + 12 \text{ ft} = 12.19 \text{ ft}
\]

The force of 21,200 lb acts 12.19 ft below the water surface and is perpendicular to the plug surface as shown.
**Pressure Prism:**

1. \[ F_{R1} = \rho h_1 (h_2 - h_1) \frac{L}{2} \]
   \[ y_{R1} = \frac{(h_2 - h_1)}{2} \]

2. \[ F_{R2} = \gamma (h_2 - h_1) \frac{L}{2} \]
   \[ y_{R2} = \frac{(h_2 - h_1)}{3} \]

\[ \theta (h_2 - h_1) \]

\[ \rightarrow F_R = F_{R1} + F_{R2} \]
\[ F_{R,yA} = F_{R1,yA} + F_{R2,yA} \]

**Example:** 2.58 → S.I

| 3m | Y=0 |
| 2m | 1.5m into page |
| 1m | |

Net Force on Gate:
1. Axis shown in figure.
Force on AB:

\[ F_A = \gamma h_A = 9.81 \frac{m}{s^2} \cdot 1000 \frac{kg}{m^3} \cdot 3m = 29430 \text{ Pa} \]

\[ F_B = \gamma h_B = (9.81 \frac{m}{s^2}) \left( \frac{1000 \frac{kg}{m^3}}{m^3} \right) (4m) = 39240 \text{ Pa} \]

Load (D):

\[ F_1 = F_A \cdot L_{AB} \cdot D = 29430 \text{ Pa} \cdot 1.414m \cdot 1.5m = 62430 \text{ N} \]

\[ F_2 = (F_B - F_1) \frac{1}{2} \cdot L_{AB} \cdot D = (39240 - 29430) \frac{1}{2} \cdot 1.414 \cdot 1.5 = 10403.505 \text{ N} \]

Line of Action:

\[ l: \text{ length along AB} \]

\[ (F_1 + F_2) l = F_1 l_1 + F_2 l_2 = (62430) \frac{1}{2} L_{AB} + (10403.505) \frac{2}{3} L_{AB} \]

\[ l = 0.7408 \text{ m} \]
Force on BC:

\[ p = \rho h_B = \rho_B = 39240 \text{ Pa} \]

\[ F_R = \rho_B L_{BC} D \]
\[ = (39240)(1)(1.5) \]
\[ = 58860 \text{ N} \]

Point of action: \( \frac{1}{2} L_{AB} \)

Free Body Diagram:

Use \( \Sigma F_y = \Sigma F_x = \Sigma M_a = 0 \) to find reactions.
Use your statics.