Bernoulli Equation I

Learning Objectives:
- Define a "streamline" and identify streamlines in diverse flows
- List the assumptions required to derive or apply the Bernoulli equation
- Express the Bernoulli equation along and normal to a streamline

Motivational Question:
- Why is it hard to breath in the gap near Pie-R Square when there is a strong wind?
Movies

- Kinematics > Streamlines
- Videos: 120, 121, and 123 (two movies)

[Click on video to play]
CVEN 311 - Fluid Dynamics
Scott A. Socolofsky
Post Objectives to screen.
Bring Fluids DVD.

Show videos:
Kinematics >
Streamlines
  120,
  121,
  123 (2 movies).

Streamlines:
Lines drawn in a flow field that are everywhere tangent to the velocity vector.

In steady flow (no time variation)
• Particle trajectories are due to velocity the particle experiences.
• Velocity at a point does not change with time.
⇒ Particles follow streamlines in steady flow.
Equation of Motion:
We derived
\[-\nabla p = \rho \ddot{\mathbf{a}} + \dot{\mathbf{r}}\]
when shear stress is absent.
(e.g. fluid moves as a bulk fluid: \(\partial u/\partial y = 0\)).

In general, forces acting on fluid particle:
- Pressure \(P\) \(\gg\) Dominant for many flows.
- Gravity \(\gg\) Friction (viscosity)

Define: Inviscid/perfect fluid \(\mu = 0\).

Assumptions for Bernoulli Equation:
1.) Viscous effects are negligible \((\mu = 0; \text{"far" from solid boundaries})\)
2.) Flow is steady \((\frac{\partial}{\partial t} = 0)\)
3.) Flow is incompressible \((\rho = \text{const. along a streamline})\)
4.) Derive equation valid only along a streamline.
Equation of Motion:

Note: \( \text{coordinate: } z = z \)

\[
\nabla z = \frac{\partial z}{\partial x} \hat{i} + \frac{\partial z}{\partial y} \hat{j} + \frac{\partial z}{\partial z} \hat{k}
\]

Then:

\[-\nabla p - \nabla z = \rho \ddot{a} \]

(for any fluid particle)

s-direction Along Streamline:

\[-\frac{d}{ds}(\rho + \nabla z) = \rho \ddot{a} \]

Acceleration:

\[a_s = \left. \frac{dv}{dt} \right|_{\text{along } s} = \frac{dv}{ds} \frac{ds}{dt} = v \frac{dv}{ds}\]

Note:

\[\frac{dv^2}{ds} = v \frac{dv}{ds} + v \frac{dv}{ds} = 2v \frac{dv}{ds}\]

Then:

\[a_s = \frac{1}{2} \frac{dv^2}{ds}\]

Substitute:

\[-\frac{d}{ds}(\rho + \nabla z) = \rho \frac{1}{2} \frac{dv^2}{ds}\]
Bernoulli Equation Along Streamline:
\[ \frac{1}{2} s \frac{d}{ds} \left( v^2 + p + \frac{\rho}{2} \right) = 0 \]

since \( g = \text{const} \) along \( s \)

\[ \frac{d}{ds} \left( p + \frac{\rho v^2}{2} + \frac{\rho}{2} \right) = 0 \]

Integrate:

\[ p + \frac{\rho v^2}{2} + \frac{\rho}{2} = C \]

\[ \frac{\rho}{g} + \frac{v^2}{2g} + z = C \]

For steady, inviscid, incompressible, along \( s \):

\[ \frac{\partial}{\partial t} = 0 \]

\[ \nu = 0 \]

\[ g = \text{const} \]

\( n \)-direction normal to Streamline:

\[ \frac{\partial}{\partial n} (-p + \rho \frac{\partial z}{\partial n}) = g a_n \]

\( R \): local radius of curvature.

\( a_n \): centrifugal force.

\[ a_n = \frac{v^2}{R} \]

\[ \frac{\partial}{\partial n} (p + \rho \frac{\partial z}{\partial n}) + g \frac{\rho v^2}{R} = 0 \]
Bernoulli Equation Normal to Streamline:

\[ \frac{\partial}{\partial n} \left( \rho + \frac{1}{2} \rho v^2 \right) = -s \frac{V^2}{R} \]

\[ \int \partial \left( \rho + \frac{1}{2} \rho v^2 \right) = -s \int \frac{V^2}{R} \, dn \]

Integrate:

\[ \frac{\rho}{\rho} + s \int \frac{V^2}{R} \, dn + \frac{1}{2} \int \frac{V^2}{R} \, dn + z = C \]

Need to know \( V \) to complete integral.

For steady, inviscid, incompressible flow taken normal to a streamline.
Breathing in Windy Gap:

at A: \( p_A = p_{atm} \)
\( v_A \ll v_B \)

1) Identify streamline AB.
2) Apply Bernoulli Eq along AB
\[ p_A + \frac{1}{2}v_A^2 + \frac{1}{2}g\Delta z = p_B + \frac{1}{2}v_B^2 + \frac{1}{2}g\Delta z \]
\( z_A = z_B \)
\[ p_A - p_B = \frac{\rho}{2} (v_B^2 - v_A^2) \]
\( \geq 0 \)
\( p_B < p_A \Rightarrow \) low pressure in gap.
If velocity high enough, difficult to breath in.