Bernoulli Equation II

Learning Objectives:
- Apply Bernoulli's equation along a streamline to solve appropriate problems
- Give the physical interpretation of each term in the Bernoulli equation along a streamline

Motivational Question:
- What is the exit velocity for the fountain jet across from the College Station Hilton?
Example 3.22: Hand in the wind:

- Force on hand.

Flow field:

\[ V_A = V_{air} \]

\[ V_A = 0 \quad \text{Stagnation point.} \]

Streamline stops at B \((V_B = 0)\) and splits into 2 streamlines.

Apply Bernoulli Eqn.:

Along AB:

\[ \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + Z_A - \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + Z_B \]

0 atm. pressure.

\[ Z_A = Z_B \]

\[ \frac{p_B}{\gamma} = \frac{1}{2} V_B^2 \]

Stagnation point.

Force on hand:

\[ F = p_B A = \frac{1}{2} V_B^2 A \]

Let \( A = 4'' \text{ by 7''} \), then

- Car @ 30 mph \( V_A = 44 \text{ ft/s} \)
- Car @ 100 mph \( V_A = 147 \)
- Plane @ 400 mph \( V_A = 587 \)
- Jet @ 800 mph \( V_A = 1173 \)

\[ F = 0.45 \text{ lb} \]

\[ 5 \text{ lb} \]

\[ 80 \text{ lb} \]

\[ 318 \text{ lb} \]
Physical Interpretation of Bernoulli:
\[
\frac{\rho}{s^2} + \frac{V^2}{2g} + y = c \quad \text{along} \quad \xi.
\]

Each term dimension \( L \rightarrow \)
Different kinds of heads.

\[
\frac{V^2}{2g} \rightarrow \text{Energy/unit weight} \rightarrow
\]
Different forms of energy.

1) \( \frac{p}{\rho g} \) : pressure head. Height of column of fluid yielding \( p \).

2) \( \frac{V^2}{2g} \) : velocity head. Vertical distance fluid must fall freely to reach \( V \) from rest.

3) \( z \) : elevation head. Potential energy.

Exit Velocity of CS Fountain:

1) Identify streamline \( AB \). (see sketch)

\[
A \quad \text{datum:} \quad z = 0
\]

2) Apply Bernoulli Eqn. along \( AB \):
\[
\frac{\rho_A}{s^2} + \frac{V_A^2}{2g} + z_A = \frac{\rho_B}{s^2} + \frac{V_B^2}{2g} + z_B
\]

0: datum \( 0: \) Stops at top.
Pressures: Close-up of nozzle.

Segment \( ab \) normal to \( s \):

\[
\rho_a + \int \frac{V_a^2}{2g} \, dn + \rho_{in} = \rho_b + \int \frac{V_b^2}{2g} \, dn + \gamma_{ab}
\]

0: \( R = \infty \)

\( \rho_a = \rho \)

\( a = 2 \)

\( b = 2 \)

\( \rho_{in} = \rho_{atm} \)

\( V_a = 0 \)

\( \Rightarrow \rho_a = 0 \).

Back to Fountain Bernoulli Eqn:

\[
\frac{p_a}{\rho g} + \frac{V_a^2}{2g} = \frac{p_b}{\rho g} + h
\]

0: open to atmosphere

0: open to atmosphere

\[
V = \sqrt{2gh}
\]

if \( h = 30 \, \text{ft} \):

\[
V = 43.93 \, \text{ft/s}
\]

\( = 30 \, \text{mph} \).