Stagnation and Dynamic Pressures

Learning Objectives:

- Explain the concept of the stagnation pressure
- Define the static, dynamic and total pressures
- Explain how a pitot tube measures velocity
- State the conservation of mass for a steady, incompressible flow

Motivational Question:

- Where is the highest pressure in a flow field?
Stagnation Pressure: largest pressure obtainable along a streamline.

\[ V_B = 0 \] since fluid is stagnant in tube.

Bernoulli Eq\textsuperscript{th} along \( AB \):

\[ p_A + \frac{1}{2} \rho V_A^2 + \rho g z_A = p_B + \frac{1}{2} \rho V_B^2 + \rho g z_B \]

\[ z_A = z_B \quad V_B = 0 \quad z_B > z_A \]

\[ p_B = p_A + \frac{1}{2} \rho V_A^2 \]

\( \downarrow \) Stagnation pressure.

Static, Dynamic & Total Pressure:

\[ \rho + \frac{1}{2} \rho V^2 + \rho g z = c \quad \text{along } s. \]

All have units of pressure.

\( \rho g z \): static or hydrostatic pressure (recall Ch. 2).

\( \frac{1}{2} \rho V^2 \): dynamic pressure

\( \rho \): thermodynamic pressure

Pressure measured by a probe moving along with fluid.

\( c \): total pressure.

\( \Rightarrow \) constant along \( s \). Pressure can change forms, but total pressure is conserved.
Pitot-static Tube:

How can we measure $\frac{V^2}{2g}$?

$\rho_A, V_A$ free stream

$P_B = $ stagnation pressure

$P_B = \rho_A + \frac{1}{2} \rho V_A^2$

$P_c = P_A$ since $V_c = V_A$

$P_D = P_c$

$\therefore V_A = \sqrt{2\left(P_B - P_D\right)/\rho}$

See pitot-tubes on p. 109 in textbook.

Design issues:

Free-stream pressure taps at C require fabrication care.

- $P_c > P_A$ (blockage)
- $P_c < P_A$ (acceleration)
- $P_c = P_A$ (good design)
Conservation of Mass:

\[ m_i = \text{mass flux into container} \]
\[ = \int_A \rho \mathbf{V} \cdot \hat{n} \, dA \]
\[ \text{unit normal of surface of opening.} \]
If \( V = \text{const.} \), \( g = \text{const.} \),
\[ = g V_1 A_1 \]

Continued.

\[ m_2 = \text{mass flux out of container.} \]
\[ = g V_2 A_2 \quad (V = \text{const.}) \]

At steady state, mass in container is constant.

\[ \frac{dM}{dt} = m_2 - m_1 = 0 \]
\[ \Rightarrow g V_1 A_1 = g V_2 A_2 \]
If \( g = \text{const.} \),
\[ \Rightarrow V_1 A_1 = V_2 A_2 \]