Examples

Learning Objectives:
- Apply conservation of mass with the Bernoulli equation to solve appropriate problems
- Identify regions of possible cavitation in pipe network problems

Motivational Question:
- How can you design a tank that will discharge at a constant rate even as the water level in the tank is dropping?
Pressure Variation across Jets:

We already saw that if $S$ is parallel with $R \rightarrow \infty$, then $p$ inside jet equals $p$ at edge of jet.

$\Rightarrow p_B = 0 \text{ : atmospheric.}$

Also, for large tanks, $V_A = 0$.

Other Geometries:

Find region with $R \rightarrow \infty$.

Area given by $d_j$

$C_c = \frac{A_j}{A_h} \sim 0.5 \text{ to } 1.0$

$l$: usually small so that we neglect $l$, but include $C_c$. 
Draining Tank:

Which tank drains faster?

1) Streamlines $AB^E_{EP}$ in sketch; datum.

2) Bernoulli Eqn to tank 1.

\[
\frac{h_1}{g} + \frac{V_A^2}{2g} + Z_A = \frac{h_2}{g} + \frac{V_B^2}{2g} + Z_B
\]

$0 \quad 0 \quad h_1 \quad 0$ \hspace{1cm} $0 \quad \text{free jet}$

Continued:

\[
V_B = \sqrt{2gh_1}
\]

3) Tank 2:

\[
\frac{h_2}{g} + \frac{V_B^2}{2g} + Z_C = \frac{h_3}{g} + \frac{V_B^2}{2g} + Z_D
\]

$0 \quad 0 \quad h_1 \quad 0 \quad -h_2$

\[
V_B = \sqrt{2g(h_1+h_2)}
\]

⇒ Extending hose below tank causes higher exit velocity.

(Ignoring friction.)

⇒ We could move hose to get lower velocity if above outlet.
Flow Constriction = Flow Meter:

\[ h_1 \]

Datum

\[ \Delta h \]

\[ A_1, A_2 \text{ known} \]
\[ Q \text{ unknown} \Rightarrow V_1, V_2 \text{ unknown} \]
\[ \Delta h \text{ known} \]

Pressure taps at 0, 1, 2 measure \( p_1, p_2 \)

1. Apply Bernoulli Eqn.:

Streamline in sketch.
Datum in sketch.

\[
\frac{p_1}{\frac{\gamma}{g}} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\frac{\gamma}{g}} + \frac{V_2^2}{2g} + z_2
\]

\[
\frac{V_1^2 - V_2^2}{2g} = \frac{p_2 - p_1}{\frac{\gamma}{g}} - \Delta h
\]

- \( \Delta h \)
Bernoulli Equation Gave:
\[ V_A^2 - V_B^2 = 2g (-\Delta h - h_i) \]

Conservation of Mass:
\[ Q = V_A A_A = V_B A_B \]
\[ \Rightarrow V_A = \frac{Q}{A_A}, \quad V_B = \frac{Q}{A_B} \]

Substitute into Bernoulli Eq.:
\[ \frac{Q^2}{A_A^2} - \frac{Q^2}{A_B^2} = 2g (-\Delta h - h_i) \]

Multiply by -1:
\[ Q^2 \left( \frac{1}{A_B^2} - \frac{1}{A_A^2} \right) = 2g (\Delta h + h_i) \]

Collect common denominator:
\[ Q^2 \left( \frac{A_A^2 - A_B^2}{A_B^2 A_A^2} \right) = 2g (\Delta h + h_i) \]

Solve:
\[ Q = \sqrt{2g (\Delta h + h_i)} \frac{A_B^2 A_A^2}{\left( A_A^2 - A_B^2 \right)} \]
Marriott Bottle:

\[ p = p_{\text{atm}} \]

\[ v = \sqrt{2g \Delta h} \]

Negative gauge pressure here.

Even while water level drops.