Conservation of Mass

Learning Objectives:
- Find appropriate control volumes to solve fluid flow problems
- Apply conservation of mass to a control volume
- Work with fixed, moving, or deformable control volumes

Motivational Question:
- How do I find an appropriate control volume?
Conservation of Mass:

Time rate of change of the system mass = 0

\[ \frac{DB}{Dt} = 0 = \frac{2}{\partial t} \int_{\Omega} \rho \, d\Omega + \int_{S_{cs}} \rho \vec{V} \cdot \hat{n} \, dA \]

- change in mass in \( \Omega \)
- fluxes through control surface.

Key Points:
1) Valid for compressible/incompressible flow
2) Did not assume inviscid.

Continuity Equation:

\[ \frac{2}{\partial t} \int_{\Omega} \rho \, d\Omega + \int_{S_{cs}} \rho \vec{V} \cdot \hat{n} \, dA = 0 \]

Real flows:
Representative velocities:

Make same mass flux
\[ \int_{A} \rho \vec{V} \cdot \hat{n} \, dA = \int_{A} \rho \vec{V} \cdot \hat{n} \, dA \]

\[ \vec{V} = \frac{\int_{A} \rho \vec{V} \cdot \hat{n} \, dA}{\int_{A} \rho \, dA} = \bar{V} \text{ for uniform flows.} \]
Mass Fluxes:

\[ \text{in} = \int_A \hat{V} \cdot \hat{n} \, dA \]

uniform \( f \) and \( A \)

\[ \int_A f \, dA = f \int_A A = f \cdot Q \]

CT with 1 inlet and 1 outlet steady:

\[ f_1 V_1 A_1 = f_2 V_2 A_2 \]

if \( f_1 = f_2 \):

\[ V_1 A_1 = V_2 A_2 \]

Examples:

1. Fixed CT
2. Moving CT
3. Deformable CT
5.2 Various types of attachments can be used with the shop vac shown in Video VS.2. Two such attachments are shown in Fig. P5.2—a nozzle and a brush. The flowrate is \( Q_1 \).\( ^3 \)\( \text{ft}^3/\text{s} \). (a) Determine the average velocity through the nozzle entrance, \( V_n \). (b) Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to \( V_b \) along the length of the bristles as shown in the figure. Determine the value of \( V_b \).

(a) \( Q_1 = Q_2 \) where \( Q_2 = \frac{\pi}{3} \) \( \text{ft}^3 \)

Thus,

\[
A_1 V_1 = Q_2 \quad \text{or} \quad V_1 = V_n = \frac{1}{\pi} \frac{\text{ft}^3}{(\frac{3}{4} \text{ ft})^2}
\]

so

\[
V_n = 45.8 \frac{\text{ft}}{\text{s}}
\]

(b) \( Q_3 = Q_4 \) where \( Q_4 = \frac{\pi}{3} \) \( \text{ft}^3 \) and \( Q_3 = \bar{V}_3 A_3 \) where

\[
\bar{V}_3 = \text{average velocity at (3)} = \frac{1}{\pi} V_b \quad \text{and}
\]

\[
A_3 = \pi D_b h_3
\]

Thus,

\[
\frac{1}{\pi} V_b \left[ \pi \left( \frac{3}{8} \text{ ft} \right) \left( \frac{3}{8} \text{ ft} \right) \right] = \frac{\pi}{3} \frac{\text{ft}^3}{\text{s}} \quad \text{or}
\]

\[
V_b = 20.4 \frac{\text{ft}}{\text{s}}
\]
3. Deformable CA.

Draining Tank:

\[
\frac{\partial}{\partial t} \int_{V_0} \rho \, dV + \int_{\partial V} \rho \mathbf{v} \cdot \mathbf{n} \, dA = 0
\]

\[
\text{const } \rho
\]

\[
\int_{V_0} \rho \, dV = V_0
\]

\[
\frac{\partial V}{\partial t} + \frac{\partial}{\partial x} (V \mathbf{v}) = 0\]

\( V = \text{const.} \)

relate \( V \) to \( h \):

\[ V = LW \cdot h \]

\( LW \frac{\partial h}{\partial t} = V_{in} A_{in} - V_{out} A_{out} \)

\( 0 \): no inflow

needs Bernoulli

Bernoulli along \( AB \):

\[ \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + \frac{h_A}{g} = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + \frac{h_B}{g} \]

\( 0 \): atm

\( 0 \): if \( LW \gg a \)

\( 0 \): atm

\( h(t) \)

\( 0 \): datum

\( \mathbf{v} \): free jet
\[
V_{out} = \sqrt{2gh(t)}
\]

Collecting:

\[
\frac{dh}{dt} = -\frac{an\sqrt{2g}h(t)}{LW}
\]

\[
\int_{h_0}^{h(t)} \frac{dh}{\sqrt{h}} = \int_{0}^{t} -\frac{an\sqrt{2g}}{LW} \, dt
\]

\[
2h^{1/2} \bigg|_{h_0}^{h(t)} = -\frac{an\sqrt{2g}}{LW} \cdot t
\]

\[
2\sqrt{h} = 2\sqrt{h_0} - \frac{an\sqrt{2g}}{LW} \cdot t
\]

When does \( h = 0 \)?

\[
t = \frac{2\sqrt{h_0} \cdot LW}{an\sqrt{2g}}
\]