Momentum Equation Examples

Learning Objectives:
- Apply the correct sign convention for the conservation of linear momentum
- Apply the momentum equation in diverse geometries
- Identify reaction forces of supports in fluid flow systems from RHS of linear momentum equation

Motivational Question:
- If the wind from hurricane Ike broke windows in downtown Houston, how strong was the wind?

TEXAS A&M ENGINEERING
Example S.28.

\[ p_1 = 210 \text{kPa} \]
\[ p_2 = 165 \text{kPa} \]
\[ A = 9000 \text{mm}^2 \]
\[ V = 15 \text{m/s} \]

Because there is no \( v \) or \( p \) acting in \( y \)-dir:
\[ R_y = 0 \]

**CV:** contains fluid inside pipe.

**FBD**

\[ v \]
\[ p_1 \]
\[ R_x \]
\[ p_2 \]

\[ R_x = -pAV(V + V) - p_1A - p_2A_2 \]
\[ Q = VA \rightarrow v_1 = v_2 \]
\[ = -7420 \text{N} \]

Required to hold bend in place.
This is force from bend into fluid.
Find $R_x$ necessary to tip block:

\[ + R \sum M_0 = 0 = R_x (0.05\text{m}) - W (0.0075\text{m}) \]

\[ R_x = 0.9 \text{ N} \]

Momentum Eq:\ only need x-dir Eq:\

\[ V_1 (\theta V_1 A_1) = -R_x \]

inflow

\[ V_1 = \sqrt{\frac{R_x}{\rho A}} = \sqrt{\frac{0.9\text{N}}{1000 \text{ kg/m}^3 \cdot \pi (0.01\text{m})^2 / 4}} = 3.39 \text{ m/s} \]

\[ Q = VA = 2.66 \cdot 10^{-4} \frac{\text{m}^3}{\text{s}} \]
Section (a)

5.28 Water flows through a horizontal, 180° pipe bend as illustrated in Fig. P5.28. The flow cross section area is constant at a value of 9000 mm². The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

**Figure P5.28**

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.24) leads to

\[ R_x = 0 \]

Application of the y-direction component of the linear momentum equation yields

\[ -\rho \nu_1^2 A_1 - \rho \nu_2^2 A_2 = p_1 A_1 - p_2 A_2 \]

or

\[ R_y = \rho A_1 \nu_1 (\nu_1 + \nu_2) + p_1 A_1 + p_2 A_2 \]

Thus

\[ R_y = \left( \frac{999 \text{ kg/m}^3}{1000 \text{ mm}^3/m^3} \right) \left( \frac{9000 \text{ mm}^2}{1000 \text{ mm}^2} \right) \left( \frac{15 \text{ m}}{s} \right)^2 \left( \frac{10^5 \text{ N/m}^2}{10^2 \text{ N/m}^2} \right) \left( \frac{1000 \text{ mm}^3}{1000 \text{ cm}^3} \right) \left( \frac{1000 \text{ N/m}^2}{10^5 \text{ N/m}^2} \right) \]

\[ + \left( \frac{65 \text{ kPa}}{10^5 \text{ N/m}^2} \right) \left( \frac{1000 \text{ mm}^2}{1000 \text{ cm}^2} \right) \left( \frac{1}{1000 \text{ cm}^2/m^2} \right) \]

\[ R_y = 7420 \text{ N} \]
5.29 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V3.4 and Fig. P5.29. Determine the minimum volume flowrate needed to tip the block.

From the free body diagram of the block when it is ready to tip \( \Sigma M_y = 0 \), or

\[
R_x l_{R_x} = \frac{W h}{L}
\]

where \( R_x \) is the force that the water puts on the block.

Thus,

\[
R_x = \frac{W h}{L} = \frac{6N \left( \frac{0.015\text{m}}{0.050\text{m}} \right)}{0.050\text{m}} = 0.90N
\]

For the control volume shown the \( x \)-component of the momentum equation

\[
\oint \rho \mathbf{V} \cdot dA = \Sigma F_x
\]

becomes

\[
V_i \rho (-V_j) A_i = -R_x \quad \text{or} \quad V_i = \frac{R_x}{\rho A_i}
\]

Thus,

\[
V_i = \sqrt{\frac{0.9N}{\left(999 \text{ kg/m}^3\right) \frac{\pi}{4} (0.01\text{m})^2}} = 3.39 \text{ m/s}
\]

Hence,

\[
Q = A_i V_i = \frac{\pi}{4} (0.01\text{m})^2 (3.39 \text{ m/s}) = 2.64 \times 10^{-4} \text{ m}^3/\text{s}
\]