Energy Equation

Learning Objectives:
• Explain the first law of thermodynamics
• Apply the sign conventions correctly for the conservation of energy equation in a fluid flow problem
• Select appropriate control volumes for applying the energy equation

Motivational Question:
• What is the power output from a water mill?
Energy Equation:

1st Law of Thermodynamics

Time rate of increase of the total stored energy in a system

\[ \frac{d}{dt} \int e \, dV = (Q_{\text{net, in}} + W_{\text{net, in}})_{\text{sys}} \]

\[ e = \frac{\text{internal } E}{\text{mass}} + \frac{K E}{\text{mass}} + \frac{P E}{\text{mass}} \]

\[ = \bar{u} + \frac{v^2}{2} + gz \]

\( Q \) : + going into system.

\( W \) : - when system does work on environment.

Apply RTT: \( \xi B = g e \)

\[ \frac{d}{dt} \int v \, d\tau + \int \vec{e} \cdot \vec{V} \, d\tau = (Q_{\text{net, in}} + W_{\text{net, in}})_{\text{cv}} \]

Radiation,
Conduction,
Convection

\[ = 0 \text{ for adiabatic process.} \]
\[ W - \text{term:} \]

1. Rotary devices. \( W = \omega T \)  
   \( \omega \) \( \rightarrow \) torque component  
   rot.  
   Frequency  
   along axis.

2. Work by fluid stresses. \( W = \int F_{\text{stress}} \cdot dV \)

\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} : \)  
normal stresses.  
\( \rightarrow p. \)

All others = shear stresses. 
\( \cdots \)  
\( \text{p. 232} \)

\[ W_{\text{stress}} = \int -p \nabla \cdot \vec{u} dA + \int (\text{shear stress}) \cdot \nabla \cdot dA \]

\( \vec{u} \) pushes on environment. 
\( \Rightarrow \) move to LHS of Eqn.

\[ \text{Summary:} \]
\[ \frac{d}{dt} \int \epsilon \rho_0 dV + \int (\dot{\vec{u}} + \frac{\dot{\rho}}{\rho} + \frac{\nabla^2}{2} + q_2) \rho \nabla \vec{u} dA = \]
\[ Q_{\text{net}} + W_{\text{shaft}} \]
Steady, uniform flow:

\[
\frac{\partial}{\partial t} = 0 \quad ; \quad v = \text{const across } A.
\]

\[
\dot{m} \left[ \hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left( \frac{\rho}{\rho_0} \right)_{\text{out}} - \left( \frac{\rho}{\rho_0} \right)_{\text{in}} + \frac{v^2}{2} \left| \text{out} \right. - \frac{v^2}{2} \left| \text{in} + g z \right|_{\text{out}} - g z \left| \text{in} \right. \right]
\]

\[
= \dot{Q}_{\text{net}} + \dot{W}_{\text{shaft}}
\]

Enthalpy:

\[
h = \hat{u} + \left( \frac{\rho}{\rho_0} \right) \quad \text{used for compressible flow (turbines).}
\]
Example 6.91

Energy Eq in terms of head (divide by g):
\[ \frac{h_{out}}{r} + \frac{V_{out}^2}{2g} + 2z_{out} = \frac{h_{in}}{r} + \frac{V_{in}^2}{2g} + 2z_{in} + h_e - h_L \]

\[ 3m + \frac{V_{2}^2}{2g} + 0 = 1m + \frac{V_{1}^2}{2g} + 1.5m - h_L \]

\[ V_1 = V_2 \]

\[ -h_L = 0.5m \]

\[ h_L = -0.5m \]

\[ \text{indicates energy GAIN by friction} \]

\[ \Rightarrow \text{impossible,} \]

\[ \therefore \text{Assumed flow direction is wrong.} \]

Flow is Uphill
\[ h_L = 0.5m \]
An incompressible liquid flows steadily along the pipe shown in Fig. P5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.

\[
\frac{P_1}{\rho} + \frac{\sqrt{2} + z_2}{2g} = \frac{P_2}{\rho} + \frac{\sqrt{2}}{2g} + z_1 + \frac{k^2}{2} - h_\ell
\]

Thus

\[
h_\ell = \frac{P_1}{\rho} - \frac{P_2}{\rho} + z_1 - z_2 = 3\text{m} - 1.0\text{m} - 1.5\text{m} = 0.5\text{m}
\]

and since \(h_\ell > 0\), the assumed direction of flow is correct.

The flow is uphill.