### Energy Equation

**Learning Objectives:**
- State the difference between the Bernoulli Equation and the Energy Equation
- Apply the energy equation to problems involving:
  - Pipe frictional losses
  - General energy losses
  - Pump sizing
- Write the energy equation in terms of head

**Motivational Question:**
- How many HP pump does a jet ski need?
Example 5.93:

\[ \text{loss} = \frac{0.8V^2}{2} \]

\( c_d = \text{Streamline in Figure.} \)

\[ \frac{P_{\text{out}}}{g} + \frac{V_{\text{out}}^2}{2} + gZ_{\text{out}} = \frac{P_{\text{in}}}{g} + \frac{V_{\text{in}}^2}{2} + gZ_{\text{in}} + W_{\text{friction}} - \text{loss} \]

\[ 0 + \frac{V_{\text{out}}^2}{2} + g \cdot 0 = 0 + \frac{V_{\text{in}}^2}{2} + g \cdot 8' \quad 0 = 0.8V^2 \]

\[ \frac{V^2}{2}(1 + 0.8) = g \cdot 8 \]

\[ V^2 = \frac{8g}{0.8} \]

\[ V = \sqrt{\frac{32.17 \text{ft}^2}{0.9}} = 16.9 \text{ ft/s} \]

Cons. of Mass:

\[ Q = VA = 16.9 \text{ ft/s} \cdot \frac{\pi}{4} \left( \frac{3}{12} \text{ ft} \right)^2 \]

\[ Q = 0.83 \frac{\text{ft}^3}{\text{s}} \]
5.93 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. 5.93. If the friction loss between A and B is \(0.8V^2/2\), where \(V\) is the velocity of flow in the siphon, determine the flowrate involved.

To determine the flowrate, \(Q\), we use

\[ Q = AV = \pi D^2 V \quad \text{(1)} \]

To obtain \(V\) we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

\[ \frac{1}{2} V_o^2 + \frac{1}{2} g z_B = \frac{1}{2} V_a^2 + \frac{1}{2} g z_A + \text{loss}_{\text{net in}} \]

or

\[ \frac{V^2}{2} + g z_B = g z_A - 0.8 V^2 \]

Thus

\[ V = \sqrt{g(z_A - z_B)} = \sqrt{\frac{32.2 \text{ ft}}{0.9}} \left(8 \text{ ft}^2\right) = 16.9 \frac{\text{ft}}{s} \]

and with Eq. 1

\[ Q = \frac{\pi D^2}{4} \left( \frac{16.9 \text{ ft}^3}{s} \right) = 0.830 \frac{\text{ft}^3}{s} \]
Example 5.109:

\[ p_2 - p_1 + \frac{v_2^2}{2g} + z_2 = \frac{P_0}{\rho} + \frac{v_1^2}{2g} + z_1 + h_s - h_L \]

- \( P_0 \): lake
- \( P_0 \): atm
- \( z_1 \): tank
- \( z_2 \): datum

\[ h_p = \frac{P_0}{\rho} + z_2 + h_L = \frac{W_p}{Q} \]

- \( W_p = 3 \text{ hp} \)
- \( P_2 = 2 \text{ (14.7 psi)} \) \( \rightarrow \) \( 4230 \text{ lb ft}^2 \)
- \( z_2 = 20 \text{ ft} \)
- \( Q = 0.223 \text{ ft}^3/\text{s} \) \( \rightarrow \) \( h_p = 119 \text{ ft} \)

then:

\[ 119' = \frac{4230 \text{ lb ft}^2}{62.4} + 20 \text{ ft} + h_L \]

\[ h_L \leq 31.2' \text{ for system to work.} \]
A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.109 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

\[ \frac{p_1}{g} + z_1 + \frac{v_1^2}{2g} + h_p - h_2 = \frac{p_2}{g} + z_2 + \frac{v_2^2}{2g}, \text{ where } p_1 = 0, \ z_1 = 0, \ v_1 = 0, \text{ and } z_2 = 20 ft. \]

Thus,

\[ h_p = h_2 + \frac{p_2}{g} + z_2. \]

Also,

\[ Q = \frac{(1000 \text{ gal})}{(10 \text{ min})} \left( \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 0.223 \text{ ft}^3 \text{ s}^{-1} \]

so that,

\[ h_p = \frac{\text{Wp}}{gQ} = \frac{(3 \text{ hp})(550 \text{ ft} \text{ lb/s})}{(62.4 \text{ lb} \text{ ft/s})}(0.223 \text{ ft}^3 \text{ s}^{-1}) = 119 \text{ ft}. \]

**(a)** If \( p_2 = 2 \text{ atm} = 2(14.7 \frac{\text{lb}}{\text{in}^2}) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) = 4230 \frac{\text{lb}}{\text{ft}^2}, \) then from Eq.(1)

\[ h_p = h_2 + \frac{4230 \text{ lb}}{62.4 \text{ lb} \text{ ft/s}} + 20 \text{ ft} = h_2 + 87.8 \text{ ft}. \]

Thus, if

\[ h_2 \leq h_p - 87.8 \text{ ft} = 119 \text{ ft} - 87.8 \text{ ft} = 31.2 \text{ ft}, \] the given pump will work for \( p_2 = 2 \text{ atm}. \)

**(b)** If \( p_2 = 3 \text{ atm} = 6.350 \frac{\text{lb}}{\text{in}^2}, \) then

\[ h_p = h_2 + \frac{6350 \text{ lb}}{62.4 \text{ lb} \text{ ft/s}} + 20 \text{ ft} = h_2 + 122 \text{ ft}. \]

Thus, if this pump is to work

\[ 119 \text{ ft} = h_2 + 122 \text{ ft}, \] or \( h_2 \leq -3 \text{ ft}, \) since it is not possible to have \( h_2 < 0, \) the pump will not work for \( p_2 = 3 \text{ atm}. \)
Example 5.107:

\[ Q = 1.5 \text{ ft}^3/\text{s} \]
\[ P = 10 \text{ psi} \]

Energy Eqn:

\[ \frac{\text{Flow}}{\text{Sec}} + \frac{V_{\text{入}}^2}{2g} + 2z_{\mathrm{in}} = \frac{P_{\text{in}}}{\text{Sec}} + \frac{V_{\text{in}}^2}{2g} + \frac{2h}{\text{in}} + h_s - h_L \]

0: atm 0: top 0: given 0: no loss

\[ z_2 = \frac{h}{\text{Sec}} \quad \frac{V_{\text{in}}^2}{2g} + h_s \]

\[ Q = VA \]
\[ 1.5 \frac{\text{ft}^3}{\text{s}} = V \frac{\pi}{4} \left( \frac{4}{12} \right)^2 \text{ft}^2 \rightarrow V = 17.2 \frac{\text{ft}}{\text{s}} \]

Solve for \( h_s \):

\[ h_s = z_2 - \frac{P_{\text{in}}}{\text{Sec}} - \frac{V_{\text{in}}^2}{2g} \]
\[ = 60 \text{ ft} - 10 \frac{\text{lb}}{\text{in}^2} \cdot \frac{1.5 \text{ ft}^2}{\text{in}^2} - \frac{(17.2 \frac{\text{ft}}{\text{s}})^2}{2 \left( 32.17 \frac{\text{ft}}{\text{s}^2} \right)} \]
\[ = 32.3 \text{ ft} \]

Power:

\[ W_{\text{shaft}} = \text{YQ} h_s = \frac{62.4 \frac{\text{lb}}{\text{ft}^3}}{\text{in}^3} \cdot 1.5 \frac{\text{ft}^2}{\text{s}} \cdot (32.3 \text{ ft}) \cdot \frac{1 \text{ hp}}{550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}} \]

\[ W_s = 5.48 \text{ hp} \]
5.107 The pumper truck shown in Fig. P5.107 is to deliver 1.5 ft³/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in. diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.

To solve this problem we first use the energy equation (Eq. 5.85) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get $h_s$ or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

\[
\frac{P_1}{P} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{P} + \frac{V_2^2}{2g} + z_2 + h_s
\]

\[h_s = z_2 - z_1 - \frac{P_2}{P} - \frac{V_1^2}{2g} \]

\[U = \frac{Q}{A} = \frac{Q}{\pi d^2 / 4} = \frac{(5 \text{ ft}^3)(4)}{\pi (4 \text{ in.})^2} = 17.2 \text{ ft}^3 / \text{s} \]

\[h_s = 60 \text{ ft} + \frac{(10 \text{ lb} / \text{in.})(1.14 \text{ in.}^2)}{(62.4 \text{ lb} / \text{ft}^3)} - \frac{(17.2 \text{ ft}^3 / \text{s})^2}{2(32.2 \text{ ft}^3 / \text{s})} \]

\[h_s = 32.3 \text{ ft} \]

\[W_{\text{shaft net in}} = \gamma Q h_s = \left(62.4 \text{ lb} / \text{ft}^3\right) \left(1.5 \text{ ft}^3 / \text{s}ight) \left(32.3 \text{ ft} \right) / 5 \text{ hp} = 5.48 \text{ hp} \]