Practical Physical Modeling

Learning Objectives:
- Explain what is meant by a distorted model
- Derive model length and velocity scales from non-dimensional numbers
- Give the critical non-dimensional numbers for:
  - Closed conduit flow
  - Flow past submerged bodies
  - Open channel flows

Motivational Question:
- How do we build an accurate model of a pump station?
Example: Las Vegas WWTP Expansion.

move fluid from low area to primary clarifiers.

Flow Types:

Each pump riser is a closed-pipe flow:

1. Geometric scales $l_1, l_2, \ldots, l_n$ 
   $(D, L, W, h, \text{etc.})$
   
   $T_i = \frac{l_i}{l} \rightarrow$ requires geometric similarity.

2. Roughness:
   
   $\frac{E_m}{D_m} = \frac{E}{D} \rightarrow$ model must be smoother.
   
   $\rightarrow$ shape of roughness must also scale
   
   $\rightarrow$ Nearly impossible.
   
   $\rightarrow$ OK if friction not too important.
3. Viscous Effects:

\[ \text{Re}_m = \frac{\text{Re}}{\text{Re}_m} \rightarrow \text{Desire Re-number similarity.} \]

\[ \frac{V_m l_m}{V_m} = \frac{V l}{V} \]

if we have water in model & prototype

\[ V_m = \nu \]

\[ L \rightarrow V_m = \frac{l}{l_m} \nu \rightarrow \text{Model velocity in pipe must be FASTER than prototype.} \]

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Flow over triangles is submerged body flow:

Very similar to pipe flow.

1. Geometric Similarity

\[ \frac{l}{l_m} \]

2. Roughness difficult but geometric.

3. \( \text{Re}_m = \text{Re} \)

Dependent variable often \( F_D \)

\[ \frac{\frac{F_D}{\frac{1}{2} \rho V^2 l^2}}{C_D} = \phi \left( \frac{L}{L} \right) \left( \frac{E}{E} \right) \left( \frac{V}{U} \right) \]
Wet-well trench is Open-Channel

1. Geometric Scale: \[ \lambda_L = \frac{l_m}{l} \]

2. Roughness on sides & bottom: \[ \frac{\theta_m}{l_m} = \frac{\epsilon}{l} \]

3. Viscous Effects \[ \text{Re}_m = \text{Re} \]

4. Free surface \[ \text{Fr}_m = \text{Fr} \]

\[ \frac{U_m}{\sqrt{g h_m}} = \frac{U}{\sqrt{g h}} \quad \text{if} \quad g = \text{same} \]

\[ \frac{U_m}{U} = \sqrt{\frac{h_m}{h}} \]

\[ = \sqrt{\lambda_m} \]

If \( \lambda_m = 1:10 \), \( \lambda_v = \sqrt{1:10} = 1:3.16 \)

What about flow rate:

\[ Q_m = V_m A_m \sim V_m L_m^2 \]

\[ \frac{Q_m}{Q} = \frac{V_m L_m^2}{V L^2} = \lambda_v \cdot \lambda_L^2 \]

\[ = \sqrt{\frac{1}{10}} \cdot \left( \frac{1}{10} \right)^2 \]

\[ = \frac{1}{316} \]

So flow rates are not enormous

\[ \Rightarrow \text{Fr} - \text{similarity usually ACHIEVED!} \]
Vortices Entering Pump:

Must also match Weber number:

\[ \text{We}_m = \text{We} \]

\[ \frac{L_m V_m^2 l_m}{\sigma_m} = \frac{\rho V^2 l}{\sigma} \]

if water
in both ⇒
cannot be
matched given
above constraints.

Distorted Models:

\[ \frac{\Delta p}{\rho V^2} \]

Pipe friction
factor

No longer
depends on Re.

\[ \frac{D}{\Lambda} \]

\[ \text{Re}_\infty \]

\[ \text{Re} \]

If \( \text{Re} \) requires: \( \lambda_V = \lambda_e^{-1} \)
And we choose: \( \lambda_V = \sqrt{\lambda_e} \)
The model is distorted in \( \text{Re} \).
⇒ Not matched.
This is OK if we are above \( \text{Re}_\infty \).
Hydraulic Institute Standards:

1. Model must be geometric scaled
   \[ \lambda_e = \frac{l_m}{\ell} \]

2. Model must match Fr similarity
   \[ \lambda_v = \sqrt{\lambda_e} \]

3. \( Re_m \geq 6 \cdot 10^4 \)  \( (p. 25) \)

4. \( We_m \geq 240 \)
American National Standard for
Pump Intake Design

Hydraulic
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Parsippany, New Jersey
07054-3802
www.pumps.org
A reasonably uniform axial velocity distribution in the suction flow (approaching the impeller) is assumed in the pump design, and non-uniformity of the axial velocity may cause uneven loading of the impeller and bearings.

A properly conducted physical model study can be used to derive remedial measures, if necessary, to alleviate these undesirable flow conditions due to the approach upstream from the pump impeller. The typical hydraulic model study is not intended to investigate flow patterns induced by the pump itself or the flow patterns within the pump. The objective of a model study is to ensure that the final sump or piping design generates favorable flow conditions at the inlet to the pump.

9.8.5.3 Model similitude and scale selection

Models involving a free surface are operated using Froude similarity since the flow process is controlled by gravity and inertial forces. The Froude number, representing the ratio of inertial to gravitational forces, can be defined for pump intakes as:

\[ F = \frac{u}{(gL)^{0.5}} \quad (9.8.5-1) \]

Where:

- \( u \) = average axial velocity (such as in the suction bell)
- \( g \) = gravitational acceleration
- \( L \) = a characteristic length (usually bell diameter or submergence)

The choice of parameter used for velocity and length is not critical, but the same parameter must be used in the model and prototype when determining the Froude number.

For similarity of flow patterns, the Froude number shall be equal in model and prototype:

\[ F_r = \frac{F_m}{F_p} = 1 \quad (9.8.5-2) \]

where \( m \), \( p \), and \( r \) denote model, prototype, and the ratio between model and prototype parameters, respectively.

In modeling a pump intake to study the potential formation of vortices, it is important to select a reasonably large geometric scale to minimize viscous and surface tension scale effects, and to reproduce the flow pattern in the vicinity of the intake. Also, the model shall be large enough to allow visual observations of flow patterns, accurate measurements of swirl and velocity distribution, and sufficient dimensional control. Realizing that larger models, though more accurate and reliable, are more expensive, a balancing of these factors is used in selecting a model scale. However, the scale selection based on vortex similitude considerations, discussed below, is a requirement to avoid scale effects and unreliable test results.

Fluid motions involving vortex formation have been studied by several investigators (Anwar, H.O. et al., 1978; Hecker, G.E., 1981; Padmanabhan, M. and Hecker, G.E., 1984; Knauss, J., 1987). It can be shown by the principles of dimensional analysis that such flow conditions at an intake are governed by the following dimensionless parameters:

\[ \frac{uD}{\Gamma}, \frac{u(gD)^{0.5}}{D/S}, uDv, \text{ and } \frac{u^2D}{\sigma} \]

Where:

- \( u \) = average axial velocity (e.g., at the bell entrance)
- \( \Gamma \) = circulation of the flow
- \( D \) = diameter (of the bell entrance)
- \( S \) = submergence (at the bell entrance)
- \( v \) = kinematic viscosity of the liquid
- \( g \) = acceleration due to gravity
- \( \sigma \) = surface tension of liquid/air interface
- \( \rho \) = liquid density

The influence of viscous effects is defined by the parameter \( uD/v = R \), the Reynolds number, and surface tension effects are indicated by \( u^2D/(\sigma\rho) = W_e \), the Weber number. Based on the available literature, the influence of viscous forces and surface tension on vortexing may be negligible if the values of \( R \) and \( W_e \) in the model fall above \( 3 \times 10^4 \) and 120, respectively, (Daggett, L., and Keulegan, G.H., 1974; Jain, A.K. et al., 1978).

With negligible viscous and surface tension effects, dynamic similarity is obtained by equating the parameters \( uD/\Gamma \), \( u(gD)^{0.5} \), and \( D/S \) in the model and prototype. An undistorted geometrically scaled Froude model satisfies this condition, provided the approach...
flow pattern in the vicinity of the sump, which governs the circulation, \( \Gamma \), is properly simulated.

Based on the above similarity considerations and including a safety factor of 2 to ensure minimum scale effects, the model geometric scale shall be chosen so that the model bell entrance Reynolds number and Weber number are above \( 6 \times 10^4 \) and 240, respectively, for the test conditions based on Froude similarity. No specific geometric scale ratio is recommended, but the resulting dimensionless numbers must meet these minimum values. For practicality in observing flow patterns and obtaining accurate measurements, the model scale shall yield a bay width of at least 300 mm (12 inches), a minimum liquid depth of at least 150 mm (6 inches), and a pump throat or suction diameter of at least 80 mm (3 inches) in the model.

In a model of geometric scale \( L_m \) with the model operated based on Froude scaling, the velocity, flow, and time scales are, respectively:

\[
V_r = \frac{V_m}{V_p} = L_r^{0.5} \tag{9.8.5-3}
\]

\[
Q_r = \frac{Q_m}{Q_p} = L_r^2 V_r = L_r^{2.5} \tag{9.8.5-4}
\]

\[
T_r = \frac{T_m}{T_p} = L_r/V_r = L_r^{0.5} \tag{9.8.5-5}
\]

Even though no scale effect of any significance is probable in models with geometric scales selected as described above, as a conservative procedure conforming to common practice, a few tests for the final design of a free surface intake shall be conducted at 1.5 times the Froude scaled flows, keeping the submergence at the geometrically scaled values. By this procedure, the circulation contributing to vortices would presumably be increased, resulting in a conservative prediction of (stronger) vortices. Tests at prototype velocities are not recommended, as this will distort approach flow patterns and unduly exaggerate flow disturbances (e.g., vortices) in the model.

Models of closed conduit piping systems leading to a pump suction are not operated based on Froude similarity, but need to have a sufficiently high pipe Reynolds number, \( R = uDp/v \), such that flow patterns are correctly scaled. Based on available data on the variation of loss coefficients and swirl with Reynolds number, a minimum value of \( 1 \times 10^6 \) is recommended for the Reynolds number at the pump suction.

9.8.5.4 Model scope

Selection of the model boundary is extremely important for proper simulation of flow patterns at the pump. As the approach flow non-uniformities contribute significantly to the circulation causing pre-swirl and vortices, a sufficient area of the approach geometry or length of piping has to be modeled, including any channel or piping transitions, bends, bottom slope changes, control gates, expansions and any significant cross-flow past the intake.

All pertinent sump structures or piping features affecting the flow, such as screens and blockage due to their structural features, trash racks, dividing walls, columns, curtain walls, flow distributors, and piping transitions must be modeled. Special care should be taken in modeling screens; the screen head loss coefficient in the model shall be the same as in the prototype. The head loss coefficient is a function of the screen Reynolds number, the percent open area, and the screen (wire) geometry. Scaling of the prototype screen wire diameter and mesh size to the selected model geometric scale may be impractical and improper due to the resulting low model Reynolds number. In some cases, a model could use the same screen as the prototype to allow equal loss coefficients. Scaling of trash racks bars may also be impractical and lead to insufficient model bar Reynolds number. Fewer bars producing the same total blockage and the same flow guidance effect (bar to space aspect ratio) may be more appropriate.

The inside geometry of the bell up to the bell throat (section of maximum velocity) shall be scaled, including any hub located between the bell entrance and the throat. The bell should be modeled of clear plastic or smooth fiberglass, the former being preferred for flow visualization. The outside shape of the bell may be approximated except in the case of multi-stage pumps, in which case the external shape may affect flow patterns approaching the inlet bell. The impeller is not included in hydraulic models, as the objective is to evaluate the effect of the intake design on flow patterns approaching the impeller. A straight pipe equal to the throat diameter or pump suction diameter shall extend at least five diameters downstream from the throat or pump suction.

For free surface intakes, the model shall provide up to 1.5 times the Froude scaled maximum flow per pump to evaluate potential scale effects on free surface vortices, as discussed above, and be deep enough to cover the range of scaled submergence.