Moody Diagram and Head Losses

Learning Objectives:
- Explain what major and minor losses are and what causes them
- Evaluate major losses in pipe networks
- Evaluate minor losses in pipe networks

Motivational Question:
- How do we calculate $h_L$ if it is not given—how do we do design?
The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flowrate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, \( h \), of the water tank under the assumption that (a) minor losses are negligible, (b) minor losses are not negligible.

\[
\rho_1 + \frac{V_2^2}{2g} + z_1 = \rho_2 + \frac{V_2^2}{2g} + z_2 + (\frac{fL}{D} + \Sigma K_L) \frac{V_2^2}{2g}, \text{ where } \rho_1 = 0, \ V_1 = 0, \ z_1 = 16 \text{ ft} + h, \ \text{and} \ z_2 = 0. \ \text{Thus, with } V = V_2.
\]

\[
16 + h = \frac{\rho_2}{g} + \frac{V_2^2}{2g} + (\frac{fL}{D} + \Sigma K_L) \frac{V_2^2}{2g}. \ \text{Note: } h \text{ must be no less than that with } \rho_2 = 60 \text{ psi and } Q_{\text{max}} = 1 \text{ cfs, or}
\]

\[
V_2 = V = \frac{q}{A} = \frac{1}{12} \text{ ft s}^2 = 5.09 \text{ ft s}^{-1}
\]

Hence,

\[
h = -16 + \left( \frac{60}{12.4} \right) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) + \left( 1 + \frac{f}{L} \left( \frac{h + 6 + 600 + 900}{12} \right) + \Sigma K_L \right) \left( \frac{5.09}{5} \right)^2 \left( \frac{1}{2(32.2 \text{ ft})^2} \right)
\]

or

\[
h = 122.5 + \left( 1 + \frac{f}{L} \left( \frac{1506 + h}{0.5} \right) + \Sigma K_L \right)(0.402) \text{ ft, where } h \sim \text{ft}
\]

With \( \frac{f}{L} = 0 \) and \( Re = \frac{VD}{\nu} = \frac{(5.09)(1.2)}{1.2 \times 10^{-8} \text{ ft}^2} = 2.1 \times 10^5 \text{ we obtain}
\]

\[
f = 0.0155 \text{ (see Fig. 8.20)}
\]

a) Neglect minor losses (\( \Sigma K_L = 0 \)):

From Eq. (1)

\[
h = 122.5 + \left( 1 + (0.0155) \left( \frac{1506 + h}{0.5} \right) \right)(0.402)
\]

or

\[
h = 143 \text{ ft}
\]

b) Include minor losses:

\[
\Sigma K_L = K_{\text{entrance}} + 15 K_{\text{elbow}} + K_{\text{tee}} = 0.5 + 15(0.3) + 0.2 = 5.2 \text{ (see Table 8.2, assume flanged fittings)}
\]

Thus, from Eq. (1)

\[
h = 122.5 + \left( 1 + (0.0155) \left( \frac{1506 + h}{0.5} \right) + 5.2 \right)(0.402)
\]

or

\[
h = 146 \text{ ft}
\]

Note: For this case minor losses are not very important.
8.42 Blood (assume $\mu = 4.5 \times 10^{-3}$ lb·s/ft², $SC = 1.0$) flows through an artery in the neck of a giraffe from its heart to its head at a rate of $2.5 \times 10^{-1}$ ft³/s. Assume the length is 10 ft and the diameter is 0.20 in. If the pressure at the beginning of the artery (outlet of the heart) is equivalent to 0.70 ft Hg, determine the pressure at the end of the artery when the head is (a) 8 ft above the heart, or (b) 6 ft below the heart. Assume steady flow. How much of this pressure difference is due to elevation effects, and how much is due to frictional effects?

\[
\frac{\rho_1 V_1^2}{2g} + z_1 = \frac{\rho_2 V_2^2}{2g} + z_2 + f \frac{L}{D} \frac{V_2^2}{2g}, \text{ where } V_1 = V_2 = V
\]

and:

\[
V = \frac{Q}{A} = \frac{2.5 \times 10^{-1}}{(\frac{0.2}{12})^2} = 1.146 \text{ ft/s}
\]

Thus, $Re = \frac{cVD}{\mu}$, or

\[
Re = \frac{(1.94 \frac{\text{slugs}}{\text{ft}})(1.146 \text{ ft/s})(\frac{0.2}{12})}{4.5 \times 10^{-5} \frac{\text{lb}-\text{s}}{\text{ft}}} = 823
\]

Hence, the flow is laminar with

\[
f = \frac{64}{Re} = \frac{64}{823} = 0.0778
\]

Also, $\rho_1 = \gamma h = (847 \frac{\text{lb}}{\text{ft}^3})(0.70 \text{ ft}) = 593 \frac{\text{lb}}{\text{ft}^3}$

Hence, from Eq. (1)

\[
\rho_2 = \rho_1 - \gamma (z_2 - z_1) - f \frac{L}{D} \frac{V^2}{2g}
\]

(a) With $z_2 - z_1 = 8 \text{ ft}$,

\[
\rho_2 = 593 \frac{\text{lb}}{\text{ft}^3} - (62.4 \frac{\text{lb}}{\text{ft}^3})(8 \text{ ft}) - 0.0778 \frac{10 \text{ ft}}{(\frac{0.2}{12})^2} \left(\frac{1}{2}\right) \left(1.94 \frac{\text{slugs}}{\text{ft}}\right) \left(1.146 \frac{\text{ft}}{\text{s}}\right)^2 = 593 \frac{\text{lb}}{\text{ft}^3} - 499 \frac{\text{lb}}{\text{ft}^3} - 59.5 \frac{\text{lb}}{\text{ft}^3} = 34.5 \frac{\text{lb}}{\text{ft}^3}
\]

Note: $-499 \frac{\text{lb}}{\text{ft}^3}$ is due to elevation, $-59.5 \frac{\text{lb}}{\text{ft}^3}$ is due to friction.

(b) With $z_2 - z_1 = -6 \text{ ft}$,

\[
\rho_2 = 593 \frac{\text{lb}}{\text{ft}^3} - (62.4 \frac{\text{lb}}{\text{ft}^3})(-6 \text{ ft}) - 0.0778 \frac{10 \text{ ft}}{(\frac{0.2}{12})^2} \left(\frac{1}{2}\right) \left(1.94 \frac{\text{slugs}}{\text{ft}}\right) \left(1.146 \frac{\text{ft}}{\text{s}}\right)^2 = 593 \frac{\text{lb}}{\text{ft}^3} + 374 \frac{\text{lb}}{\text{ft}^3} - 59.5 \frac{\text{lb}}{\text{ft}^3} = 908 \frac{\text{lb}}{\text{ft}^3}
\]

Note: $374 \frac{\text{lb}}{\text{ft}^3}$ is due to elevation, $-59.5 \frac{\text{lb}}{\text{ft}^3}$ is due to friction.
The pump shown in Fig. P8.75 delivers a head of 250 ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200 ft.

\[ \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_p = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \]

where \( P_1 = P_2 = 0 \), \( V_1 = V_2 = 0 \), \( z_1 = 0 \), \( z_2 = 200 \) ft, \( h_p = 250 \) ft

Thus,

\[ -f \frac{L}{D} \frac{V^2}{2g} - \sum K_i \frac{V^2}{2g} + h_p = z_2 \]

so that with \( \sum K_i \frac{V^2}{2g} = (0.8 + 4(1.5) + 5.0 + 1) \frac{V^2}{2g} = 12.8 \frac{V^2}{2g} \)

\[ \left[ -f \left( \frac{500}{0.75} \right) - 12.8 \right] \frac{V^2}{2(32.2)} + 250 = 200 \]

or

\[ (667 f + 12.8) V^2 = 3220 \]

Also, \( Re = \frac{\rho V D}{\mu} = \frac{(1.94 \frac{slugs}{ft^3})(0.75 \text{ ft})}{2.34 \times 10^{-5} \frac{lb}{ft^3} \frac{lb}{s} \frac{ft}{s}} \)

or

\[ Re = 6.22 \times 10^4 V \]

and from Fig. 8.20:

\[ f \frac{D}{L} = 0 \]

**Trial and error solution.** Assume \( f = 0.02 \)

\( V = 11.1 \frac{ft}{s} \) \( \rightarrow Re = 6.9 \times 10^3 \)

\( f \rightarrow 0.012 \neq 0.02 \)

Assume \( f = 0.012 \)

\( V = 12.4 \frac{ft}{s} \) \( \rightarrow Re = 7.7 \times 10^4 \)

\( f \rightarrow 0.0121 \approx 0.012 \)

Thus, \( V = 12.4 \frac{ft}{s} \) and

\[ \dot{W} = 8Qh_p = (62.4 \frac{lb}{ft^3})(0.75 \text{ ft})^2(12.4 \frac{ft}{s})(2.50 \text{ ft}) = 8.55 \times 10^4 \frac{ft \cdot lb}{s} \]

\[ = 8.55 \times 10^4 \frac{ft \cdot lb}{s} \times \frac{1 \text{ hp}}{550 \frac{ft \cdot lb}{s}} = 155 \text{ hp} \]

Alternatively, we could replace Eq. (3) (the Moody chart) by Eq 8.35