Learning Objectives:
- Calculate drag on objects in uniform flow.
- Estimate drag for composite objects in uniform flow.

Motivational Question:
- How far from the mouth of the Mississippi River can we expect a significant sediment plume?
9.39 Determine the drag on a small circular disk of 0.01 ft diameter moving 0.01 ft/s through oil with a specific gravity of 0.87 and a viscosity 10,000 times that of water. The disk is oriented normal to the upstream velocity. By what percent is the drag reduced if the disk is oriented parallel to the flow?

\[ \mathcal{D} = C_D \frac{1}{2} \rho U^2 A, \quad \text{where} \quad \rho = (0.87)(1.688 \text{ slugs/ft}^3) = 1.688 \text{ slugs/ft}^3 \]

and

\[ \mu = 10^4 \mu_{\text{H}_2\text{O}} = 10^4 (2.34 \times 10^{-5} \text{ lb-s/ft}^2) = 0.234 \text{ lb-s/ft}^2 \]

Thus,

\[ \text{Re} = \frac{U D}{\nu} = \frac{\rho U D}{\mu} = \frac{(1.688 \text{ slugs/ft}^3)(0.01 \text{ ft})(0.01 \text{ ft})}{0.234 \text{ lb-s/ft}^2} = 7.21 \times 10^{-4} \ll 1 \]

so that the low Re data of Table 9.4 is valid.

For the disk normal to the flow, \( C_D = \frac{20.4}{7.21 \times 10^{-4}} = 2.83 \times 10^4 \)

so that from Eq. (i)

\[ \mathcal{D} = 2.83 \times 10^4 \left( \frac{1}{2} \right)(1.688 \text{ slugs/ft}^3)(0.01 \text{ ft})^2 \frac{7.21 \times 10^{-4}}{0.01 \text{ ft}^2} = 1.88 \times 10^{-4} \text{ lb} \]

If the disk is parallel to the flow, \( C_D = \frac{13.6}{7.21 \times 10^{-4}} \)

so that

\[ \frac{\mathcal{D}_{\text{parallel}}}{\mathcal{D}_{\text{normal}}} = \frac{C_{D_{\text{parallel}}}}{C_{D_{\text{normal}}}} = \left( \frac{13.6}{20.4} \right) \frac{7.21 \times 10^{-4}}{0.01 \text{ ft}^2} = 0.667, \quad \text{a 33.3\% reduction} \]

9.40 For small Reynolds number flows the drag coefficient of an object is given by a constant divided by the Reynolds number (see Table 9.4). Thus, as the Reynolds number tends to zero, the drag coefficient becomes infinitely large. Does this mean that for small velocities (hence, small Reynolds numbers) the drag is very large? Explain.

For a given object \( C_D = \frac{C}{\text{Re}} \) (where the value of \( C \) depends on the shape of the object), provided \( \text{Re} \leq 1 \). Thus, as \( \text{Re} \to 0 \), \( C_D \to \infty \).

However,

\[ \mathcal{D} = C_D \frac{1}{2} \rho U^2 A = \left( \frac{C}{\text{Re}} \right) \frac{1}{2} \rho U^2 A \sim U \]

That is, as \( U \to 0 \) (i.e., \( \text{Re} \to 0 \)), then \( \mathcal{D} \sim U \)

Thus, does \( C_D \to \infty \) mean that \( \mathcal{D} \to \infty ? \) No.
Reynolds Number Dependence. Another parameter on which the drag coefficient can be very dependent is the Reynolds number. The main categories of Reynolds number dependence are (1) very low Reynolds number flow, (2) moderate Reynolds number flow (laminar boundary layer), and (3) very large Reynolds number flow (turbulent boundary layer). Examples of these three situations are discussed below.

Low Reynolds number flows (Re < 1) are governed by a balance between viscous and pressure forces. Inertia effects are negligibly small. In such instances the drag on a three-dimensional body is expected to be a function of the upstream velocity, \( U \), the body size, \( \ell \), and the viscosity, \( \mu \). That is,

\[
\mathcal{D} = f(U, \ell, \mu)
\]

From dimensional considerations (see Section 7.7.1)

\[
\mathcal{D} = C\mu\ell U
\]

where the value of the constant \( C \) depends on the shape of the body. If we put Eq. 9.38 into dimensionless form using the standard definition of the drag coefficient, we obtain

\[
C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 \ell^2} = \frac{2C\mu U}{\rho U^2 \ell^2} = \frac{2C}{Re}
\]

where Re = \( \rho U \ell / \mu \). The use of the dynamic pressure, \( \rho U^2 / 2 \), in the definition of the drag coefficient is somewhat misleading in the case of creeping flows (Re < 1) because it introduces the fluid density, which is not an important parameter for such flows (inertia is not important). Use of this standard drag coefficient definition gives the \( 1/Re \) dependence for small Re drag coefficients.

Typical values of \( C_D \) for low Reynolds number flows past a variety of objects are given in Table 9.4. It is of interest that the drag on a disk normal to the flow is only 1.5 times greater than that on a disk parallel to the flow. For large Reynolds number flows this ratio is considerably larger (see Example 9.1). Streamlining (i.e., making the body slender) can produce a considerable drag reduction for large Reynolds number flows; for very small Reynolds number flows it can actually increase the drag because of an increase in the area on which shear forces act. For most objects, the low Reynolds number flow results are valid up to a Reynolds number of about 1.

### Table 9.4
Low Reynolds Number Drag Coefficients (Ref. 7) (Re = \( \rho UD / \mu, A = \pi D^2 / 4 \))

<table>
<thead>
<tr>
<th>Object</th>
<th>( C_D = \mathcal{D} / (\rho U^2 A / 2) ) (for Re ( \leq 1 ))</th>
<th>Object</th>
<th>( C_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Circular disk normal to flow</td>
<td>20.4/Re</td>
<td>c. Sphere</td>
<td>24.0/Re</td>
</tr>
<tr>
<td>( U \rightarrow \square )</td>
<td></td>
<td>( U \rightarrow \square )</td>
<td></td>
</tr>
<tr>
<td>b. Circular disk parallel to flow</td>
<td>13.6/Re</td>
<td>d. Hemisphere</td>
<td>22.2/Re</td>
</tr>
<tr>
<td>( U \rightarrow \square )</td>
<td></td>
<td>( U \rightarrow \square )</td>
<td></td>
</tr>
</tbody>
</table>
9.57 The structure shown in Fig. P9.57 consists of a cylindrical support post to which a rectangular flat-plate sign is attached. Estimate the drag on the structure when a 50-mph wind blows against it.

\[ D = D_{\text{sign}} + D_{\text{post}}, \text{ where } D_{\text{sign}} = \frac{1}{2} \rho U^2 A_{\text{sign}} C_{D_{\text{sign}}} \text{ and } D_{\text{post}} = \frac{1}{2} \rho U^2 A_{\text{post}} C_{D_{\text{post}}} \]

Also, \( A_{\text{sign}} = 12 \text{ ft} \times (24 \text{ ft}) = 288 \text{ ft}^2 \) and 
\( A_{\text{post}} = 3 \text{ ft} \times (30 \text{ ft}) = 90 \text{ ft}^2 \)

From Fig. 9.28, for a thin flat plate with \( l/D \approx 0.1, C_D = 1.9 \)

Thus, \( C_{D_{\text{sign}}} = 1.9 \)

Also, for the cylinder (post), \( Re = \frac{UD}{v} \), where
\[ U = \frac{50 \text{ mi}}{\text{hr}} \times \frac{(1 \text{ hr})}{(60)^2 \text{s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} = 73.3 \text{ ft/s} \]
so that
\[ Re = \frac{(73.3 \text{ ft/s}) (3 \text{ ft})}{1.57 \times 10^{-4} \text{ ft}^2/\text{s}} = 1.40 \times 10^6 \]

Hence, from Fig. 9.21, \( C_{D_{\text{post}}} = 0.8 \)

By using the above data, Eq. (1) gives
\[ D = \frac{1}{2} \rho U^2 \left[ A_{\text{sign}} C_{D_{\text{sign}}} + A_{\text{post}} C_{D_{\text{post}}} \right] \]
\[ = \frac{1}{2} \times (0.00238 \text{ slugs/ft}^3) (73.3 \text{ ft/s})^2 \times \left[ 288 \text{ ft}^2 (1.9) + 90 \text{ ft}^2 (0.8) \right] \]
\[ D = 3960 \text{ lb} \]
Moderate Reynolds number flows tend to take on a boundary layer flow structure. For such flows past streamlined bodies, the drag coefficient tends to decrease slightly with Reynolds number. The \( C_D \sim \text{Re}^{-1/2} \) dependence for a laminar boundary layer on a flat plate (see Table 9.3) is such an example. Moderate Reynolds number flows past blunt bodies generally produce drag coefficients that are relatively constant. The \( C_D \) values for the spheres and circular cylinders shown in Fig. 9.21a indicate this character in the range \( 10^3 < \text{Re} < 10^5 \).

The structure of the flow field at selected Reynolds numbers indicated in Fig. 9.21a is shown in Fig. 9.21b. For a given object there is a wide variety of flow situations, depending

![Flow past a cylinder can take on a variety of different structures.](image)

![Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.](image)

![Typical flow patterns for flow past a circular cylinder at various Reynolds numbers as indicated in (a).](image)
<table>
<thead>
<tr>
<th>Shape</th>
<th>Reference area ( A ) ( (b = \text{length}) )</th>
<th>Drag coefficient ( C_D = \frac{8}{\pi} \frac{D}{b} \frac{U^2}{\mu} A )</th>
<th>Reynolds number ( Re = \rho U D / \mu )</th>
</tr>
</thead>
</table>
| Square rod with rounded corners | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re = 10^5 \) |
| 0                            | 2.2      | 2.0          |
| 0.02                         | 1.2      | 2.0          |
| 0.08                         | 1.5      | 1.9          |
| 0.25                         | 1.1      | 1.3          |
|                              |          |              |                    |
| Rounded equilateral triangle  | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re = 10^5 \) |
| 0                            | 1.4      | 2.1          |
| 0.02                         | 1.2      | 2.0          |
| 0.08                         | 1.5      | 1.9          |
| 0.25                         | 1.1      | 1.3          |
|                              |          |              |                    |
| Semicircular shell           | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re = 2 \times 10^4 \) |
| 0                            | 2.3      | 1.1          |
|                              |          |              |                    |
| Semicircular cylinder        | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re > 10^4 \) |
| 0                            | 2.15     | 1.15         |
|                              |          |              |                    |
| T-beam                       | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re > 10^4 \) |
| 0                            | 1.80     | 1.65         |
|                              |          |              |                    |
| I-beam                       | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re > 10^4 \) |
| 0                            | 2.05     |              |
|                              |          |              |                    |
| Angle                        | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re > 10^4 \) |
| 0                            | 1.98     | 1.82         |
|                              |          |              |                    |
| Hexagon                      | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( R/D \) | \( C_D \) | \( Re > 10^4 \) |
| 0                            | 1.0      |              |
|                              |          |              |                    |
| Rectangle                    | \( A = bD \)                                   | \begin{array}{c|c|c}
|                                | \( \phi \) | \( C_D \) | \( Re = 10^5 \) |
| \( \leq 0.1 \)               | 1.9      |              |
| 0.5                          | 2.5      |              |
| 0.65                         | 2.9      |              |
| 1.0                          | 2.2      |              |
| 2.0                          | 1.6      |              |
| 3.0                          | 1.3      |              |

**Figure 9.28** Typical drag coefficients for regular two-dimensional objects (Refs. 5, 6).

The lift coefficient is a function of other dimensionless parameters.

The Froude number, \( Fr \), is important only if there is a free surface present, as with an underwater "wing" used to support a high-speed hydrofoil surface ship. Often the surface roughness, \( \varepsilon \), is relatively unimportant in terms of lift—it has more of an effect on the drag. The Mach number, \( Ma \), is of importance for relatively high-speed subsonic and supersonic flows (i.e., \( Ma > 0.8 \)), and the Reynolds number effect is often not great. The most important
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\[
D = C_D \frac{1}{2} \rho U^2 A, \quad \text{where} \quad \rho = (0.87) (1.94 \frac{\text{slug}}{\text{ft}^3}) = 1.688 \frac{\text{slug}}{\text{ft}^3}
\]

and

\[
\mu = 10^4 \frac{\mu_{\text{H}_2\text{O}}}{10^4 (2.34 \times 10^{-5} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2})} = 0.234 \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}
\]

Thus, \( Re = \frac{UD}{\nu} = \frac{\rho UD}{\mu} = \frac{(1.688 \frac{\text{slug}}{\text{ft}^3})(0.01 \frac{\text{ft}}{2})(0.01 \frac{\text{ft}}{2})}{0.234 \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}} = 7.21 \times 10^{-4} \ll 1 \)

so that the low \( Re \) data of Table 9.4 is valid.

For the disk normal to the flow, \( C_D = \frac{20.4}{Re} = \frac{20.4}{7.21 \times 10^{-4}} = 28300 \)

so that from Eq. (1)

\[
D = 28300 (\frac{1}{2})(1.688 \frac{\text{slug}}{\text{ft}^3})(0.01 \frac{\text{ft}}{2})(0.01 \frac{\text{ft}}{2}) = 1.88 \times 10^{-4} \text{ lb}
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\[
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Thus, does \( C_D \to \infty \) mean that \( D \to \infty \)？ No.
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\[
D = D_{\text{sign}} + D_{\text{post}}, \quad \text{where} \quad D_{\text{sign}} = \frac{1}{2} \rho U^2 A_{\text{sign}} C_{D_{\text{sign}}} \quad \text{and} \quad D_{\text{post}} = \frac{1}{2} \rho U^2 A_{\text{post}} C_{D_{\text{post}}}
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Also, \( A_{\text{sign}} = 12 \text{ ft} \times 24 \text{ ft} = 288 \text{ ft}^2 \) and \( A_{\text{post}} = 3 \text{ ft} \times 30 \text{ ft} = 90 \text{ ft}^2 \)

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\]

so that

\[
Re = \frac{(73.3 \text{ ft/s}) (3 \text{ ft})}{1.57 \times 10^{-4} \text{ ft}^3 \text{s/lb}} = 1.40 \times 10^6
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Hence, from Fig. 9.21, \( C_{D_{\text{post}}} = 0.8 \)

By using the above data, Eq. (1) gives

\[
D = \frac{1}{2} \rho U^2 \left[ A_{\text{sign}} C_{D_{\text{sign}}} + A_{\text{post}} C_{D_{\text{post}}} \right]
\]

\[
= \frac{1}{2} (0.00238 \text{ slug/ft}^3) (73.3 \text{ ft/s})^2 \left[ 288 \text{ ft}^2 (1.9) + 90 \text{ ft}^2 (0.8) \right]
\]

or

\[
D = 3,960 \text{ lb}
\]