CVEN 489-501: Special Topics in Mixing and Transport Processes in the Environment

Engineering – Lectures

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Recommended Reading

Journal Articles

Journals are a major source of information on Environmental Fluid Mechanics. Major journals include *Environmental Fluid Mechanics*, published by Kluwer, the *Journal of Hydraulic Engineering* published by the American Society of Civil Engineers (ASCE), the *Journal of Hydraulic Research* published by the International Association of Hydraulic Engineering and Research (IAHR), *Limnology and Oceanography* published by the American Society of Limnology and Oceanography (ASLO), *Physics of Fluids*, published by the American Physics Society (APS), and the *Journal of Fluid Mechanics* published by Cambridge University Press, among many others. These are all available through the Texas A&M University library: newer articles are available through on-line subscriptions, and older articles (usually before 1995) are available in bound volumes in the library stacks.

Supplemental Textbooks

The material for this course is also treated in a number of other books; in particular, the following supplementary texts are recommended:


Condensed Bibliography

The following books are also recommended for in-depth study of individual topics:


Preface

Environmental Fluid Mechanics (EFM) is the study of motions and transport processes in earth’s hydrosphere and atmosphere on a local or regional scale (up to 100 km). At larger scales, the Coriolis force due to earth’s rotation must be considered, and this is the topic of Geophysical Fluid Dynamics. Sticking purely to EFM in this book, we will be concerned with the interaction of flow, mass and heat with man-made facilities and with the local environment.

This text is the first Part in a two-part book to accompany a two-semester course in Environmental Fluid Mechanics. In this Part, Mixing and Transport Processes in the Environment, passive diffusion is treated by introducing the transport equation and its application in a range of unstratified water bodies. Passive diffusion refers to mixing processes that occur due to random motions and that have no direct feedback on the dynamics of the fluid motion. The second Part, Stratified Flow and Buoyant Mixing, covers the dynamics of stratified fluids and transport under active diffusion. Active diffusion relates to mixing processes that have a direct feedback on the equations of motion due to changes in the density of the carrier fluid. This first Part is appropriate for senior level undergraduate students; whereas, the second Part is more appropriate for first-year graduate students.

The text is designed to compliment existing text books in water quality, air quality, and transport. A unique feature of this text is that most of the mathematics is written out in sufficient detail that all of the equations should be derivable (and checkable!) by the reader. This fifth edition adds more homework problems to each chapter and expands the text and explanations in each chapter.

The chapters are all organized in a similar fashion. Following the chapter heading, the first two paragraphs orient the chapter in the context of the other chapters and outline the material to be covered. In the first section of the chapter, general, background information is covered that is needed to fully understand the contents of the chapter. The middle sections develop the appropriate theory and present the mathematical derivations. The final section in each chapter presents applications of the material to engineering practice. At the end of each chapter, a summary section highlights the key points and a set of exercises are presented as possible homework problems. The book contains a single references section and index.

This book was compiled from several sources, including the lecture notes developed by Gerhard H. Jirka for courses offered at Cornell University and the University of Karlsruhe, lecture notes developed by Scott A. Socolofsky for courses taught at the University of Karlsruhe and Texas A&M University, and notes taken by Scott A. Socolofsky in various fluid mechanics courses offered at the Massachusetts Institute of Technology (MIT), the University of Colorado, and the

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Comments, questions, and corrections on this script can always be addressed per E-Mail to the address: scoolofo@tamu.edu.

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