1 Diffusion in a River

Diffusion in a river. The Rhein river can be approximated as having a uniform depth \( h = 5 \text{ m} \), width \( B = 300 \text{ m} \) and mean flow velocity \( u = 0.7 \text{ m/s} \). Under these conditions, 100 kg of tracer is injected in a slice, uniformly across the cross-section (creating an instantaneous plane source normal to the flow direction). The cloud is expected to diffuse longitudinally (in the flow direction) as a one-dimensional point source in a moving coordinate system, moving at the mean stream velocity. The river has an enhanced mixing coefficient of \( D = 10 \text{ m}^2/\text{s} \). How long does it take for the center of mass of the cloud to reach a point \( x = 15000 \text{ m} \) downstream? What is the maximum concentration that passes the point \( x \)? How wide is the cloud (take the cloud width as \( 4\sigma \)) when it passes this point?

2 Measuring Diffusion Coefficients

As part of a water quality study, you have been asked to assess the diffusion of a new fluorescent dye\(^1\). To accomplish this, you do a dye study in a laboratory tank (depth \( h = 40 \text{ cm} \)). You release the dye at a depth of 20 cm (spread evenly over the area of the tank) and monitor its development over time. Vertical profiles of dye concentration in the tank are shown in Figure 1; the \( x \)-axis represents the reading on your fluorometer and the \( y \)-axis represents the depth.

- Estimate the molecular diffusion coefficient of the dye, \( D_m \), based on the evolution of the dye cloud (Hint: estimate \( \sigma \) at the two times and use the relationship \( \sigma^2_2 - \sigma^2_1 = 2D_m\Delta t \)).
- Predict at what time the vertical distribution of the dye will be affected by the boundaries of the tank.

\(^1\)This problem is adapted from Nepf (1995).
Figure 1: Concentration profiles of fluorescent dye for two different measurement times. Refer to problem number 2.

3 Concentration profiles

Figure 2 shows four concentration profiles measured very carefully at the bottom of four different lakes. For each profile, state whether the lake bottom is a no-flux or flux boundary and describe where you think the source is located and why.

4 Image sources in a pipe

A point source is released in the center of an infinitely long round pipe. Describe the image source needed to account for the pipe walls.

References

Figure 2: Bottom concentration profiles for the four lakes in problem number 3.