Integral definitions:

\[ \overline{u} = \frac{1}{T} \lim_{T \to \infty} \int_0^T u(t) \, dt \]

or

\[ \overline{u} = \frac{1}{L} \lim_{L \to \infty} \int_0^L u(x) \, dx \]

The scale at which velocity (measurement) becomes uncorrelated:

\( l_I \): integral length scale.
\( u_I \): integral velocity scale.
\( t_I \): integral time scale.

\[ l_I = \int_0^\infty R_{uu}(\tau) \, d\tau \]
\[ t_I = \int_0^\infty R_{tt}(\tau) \, d\tau \]
Concept of Turbulent Diffusion:

Fickian diffusion is transport by random motions. Our probabilistic solution was
\[ D \propto \frac{(\Delta x)^2}{\Delta t}. \]

Turbulence is random motion. Also, the majority of mass is transported at large scales. From the correlation function, turbulent sizes (scales) are:

\[ u_I, l_I, t_I : \text{integral scales}. \]

Then, we have:
\[ D_t \propto \frac{l_I^2}{t_I} = u_I l_I \propto h: \text{limiting scale}. \]
\[ u_I \propto u_*: \text{shear velocity}. \]
\[ D_t \propto u_* h. \]

To use this "turbulent diffusion" coefficient, we need to derive a transport equation for a turbulent flow.

Reynold's Decomposition:

at \( x_0 \):

\[ u \]

\[ \overline{u}(x_0) \]

\[ u'(x_0, t) \]
Homogeneous turbulence: statistics of \( u'(x_0,t) \) are independent of space.

Isotropic turbulence: statistics of \( u', v', \) and \( w' \) are identical.

Stationary turbulence: \( \bar{u}(x_0) \) constant in time.

We may relax this a bit and let \( \bar{u}(x_0) \) vary slowly compared to \( t_x \).

Reynolds Decomposition:

\[
\begin{align*}
\ u(x_0,t) & = \bar{u}(x_0) + u'(x_0,t) \\
\end{align*}
\]

\( u' \): assumed to be a random variable.

It has a probability distribution and a correlation function and zero mean.

For the turbulent transport equation, we do an analogous decomposition for \( c' \):

\[
\begin{align*}
\ C(x_0,t) & = \bar{c}(x_0) + c'(x_0,t) \\
\end{align*}
\]

Second random variable, assumed correlated with \( u' \), and also with zero mean.

Define a time average:

\[
\overline{f} = \frac{1}{T} \int_{t}^{t+T} f(t) \, dt
\]

Example:

\[
\overline{u'} = \frac{1}{T} \int_{t}^{t+T} u'(t) \, dt = 0 \quad (\text{if } T \text{ is greater than } t_x).
\]
Properties of decomposition:

\[ \bar{u}' = 0 \quad \bar{c}' = 0 \quad \bar{u}' \bar{c}' = a \text{ non-zero number} \]

\[ \bar{u} = \bar{u}' \quad \bar{c} = \bar{c}' \quad \bar{u}' \bar{c}' = \bar{c}' \bar{u}' = 0. \]

\[ \bar{u}' \bar{u}' = a \text{ non-zero number.} \]

\[ \bar{c}' \bar{c}' = a \text{ non-zero number.} \]