Conservative System:
\[ \frac{dM}{dt} = 0 \]

Reactive System:
\[ \frac{dM_i}{dt} \neq 0 = S_i \quad \text{mass changes for constituent, source/sink term.} \]

Homogeneous Reactions: reaction occurs distributed throughout water column.

Heterogeneous Reactions: reaction occurs only at fluid boundary.

Rate laws:
\[ \frac{d[A]}{dt} = R_A \]
rate law for species A; determined experimentally.

General Form:
\[ R_i = k_i \prod_{j=1}^{n_j} C_j^{\eta_j} \]

- \( i \): # of products
- \( j \): # of reactions
- \( k \): rate constant.
- \( \eta_j \): order of reaction in species \( j \).

\[ K = \prod_{i=1}^{j} \eta_i \quad \text{overall reaction order. units of } k \text{ depend on } K. \]
Common Reaction Rate Laws.

1st Order Reaction:

\[
\frac{dc}{dt} = \pm kc
\]

\[\Rightarrow\]

assume \( k \) and \( c > 0 \).

+ : production.

- : loss.

- Examples: radioactive decay.
  die-off of bacteria in a river.

\[ k : \left[ \frac{1}{T} \right] \quad \text{‘number per time’}. \]

- Solution:

  Initial condition: \( c(0) = c_0 \).

  \[ \int \frac{dc}{c} = \int \pm k dt \quad k \text{ is always independent of time.} \]

  \[ \ln c = \pm kt + C' \]

  \[ c = c' \exp (\pm kt) \]

  \[ c(t) = c_0 \exp (\pm kt) \]

  \[ c_{1/2} = \frac{1}{2} c_0 \]

  \[ T = \frac{1}{k} \ln 2 \]

  \[ T_L = \frac{1}{k} \ln \left( \frac{c_0}{c_L} \right) \]

  \[ c(t) = c_0 \exp (-kt) \]

  \[ c(t) = c_0 \exp (+kt) \]
Die-off: (true for any reaction with sink).

Half-life: time until \( C = C_0/2 \):
\[
T_{1/2}.
\]
e-folding time: time until \( C = C_0/e \):
\[
T_e.
\]

1st order Rxn:
\[
\frac{C}{C_0} = e^{-kt}
\]

1.) \( T_{1/2} \):
\[
\frac{1}{2} = e^{-kt}
\]
\[
-kT_{1/2} = \ln \left( \frac{1}{2} \right)
\]
\[
T_{1/2} = -\frac{\ln \left( \frac{1}{2} \right)}{k} = \frac{0.69}{k}
\]

\[
T_{1/2} = \frac{0.69}{k}
\] depends on \( k \). independent of \( C_0 \)!

2.) \( T_e \):
\[
\frac{1}{e} = e^{-kt}
\]
\[
-kT_e = \ln \left( \frac{1}{e} \right)
\]

or
\[
\frac{1}{e} = \frac{1}{e^{kt_e}}
\]
\[
1 = -kt_e
\]
\[
T_e = \frac{1}{k}
\]