Example: Wastewater treatment plant.

\[ Q = 1 \frac{m^3}{s}, \quad B = 20 m, \quad h = 0.2 m \]

Swimming standard: \( 2000 \frac{\#}{100 ml} \).

Diffuser.

\( \dot{m}_{in} \): bacteria input from WWTP. = \( 5 \times 10^{10} \frac{\#}{s} \).

Steady-state.

If no die-off: concentration d/s will be constant.

Assume 1st order decay: \( k = 0.8 \frac{1}{\text{day}} \).

Find \( C(x) \) and \( x_{\frac{1}{2}} \).

Time-dependent solution:

\[ \frac{dC}{dt} = -kC \quad C(0) = C_0 \]

\[ C(t) = C_0 \exp(-kt) \]

Use Lagrangian approach: move with a packet of water.

\[ x = ut \]

\[ C \left( \frac{x}{u} \right) = C_0 \exp \left( -\frac{kx}{u} \right) \].

\( C_0 = ? \)

\( u = ? \)
\[ u = \frac{Q}{A} = \frac{Q}{Bh} \]

\[ u = \frac{1 \text{ m}^3}{20 \text{ m} (0.2 \text{ m})} = \boxed{0.25 \text{ m/s}} \]

\[ C_o = ?; \text{ use mass flow rate.} \]

\[ \dot{m} = QC \]

\[ 5 \times 10^{10} \text{ lb/s} = \frac{1 \text{ m}^3}{\text{s}} \cdot C_o \]

\[ C_o = \frac{5 \times 10^{10} \text{ lb}}{\text{m}^3} \cdot \frac{1 \text{ m}^3}{10^3 \text{ lb}} \cdot \frac{1 \text{ hr}}{10 (100 \text{ ml})} \]

\[ C_o = 5 \cdot 10^6 \text{ lb/100 ml} \]

Change units of \( k \):

\[ k = 0.8 \text{ lb/ft/hr} \cdot \frac{1 \text{ ft}}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \]

\[ k = 9.26 \cdot 10^{-6} \frac{\text{lb}}{\text{s}} \]

Downstream concentration:

\[ C(x) = 5 \cdot 10^6 \exp \left( \frac{-9.26 \cdot 10^{-6} (x)}{0.25} \right) \left[ \frac{\text{lb}}{100 \text{ ml}} \right] \]

\[ C(x) = 5 \cdot 10^6 \exp \left( -3.7 \cdot 10^{-5} x \right) \]

\[ k \left[ \frac{1}{\text{m}} \right]. \]
Where is $\frac{C}{C_0} = \frac{1}{2}$?

$$\frac{1}{2} = \exp\left(-3.7 \times 10^{-5} x \frac{1}{2}\right)$$

$$x \frac{1}{2} = \frac{-0.69}{-3.7 \times 10^{-5}} = 1.86 \times 10^4 \text{ m}$$

Concentration decreases slowly.

Why do we neglect diffusion/dispersion?

$$Pe = \frac{D}{uL} \ll \frac{D}{ux} \ll 1$$

downstream distance.

Diffusion effect:

$$q = -D \frac{dc}{dx}$$

\[ \text{with no die-off = 0.} \]

\[ \text{with die-off, may be small.} \]

In general diffusion may affect $C$, but for steady-state, affect may be small.