Dimensional Analysis:

Used to develop nondimensional relationships that describe or predict the behavior of physical processes.

Example:

→ diffusing odor.

→ L → person waiting to smell paint.

→ paint can.

→ How long will it take from when the paint can is opened until the person smells the paint.

Governing Parameters:

\( t \): time until person smells paint. \([T]\).

\( L \): distance between paint and person. \([L]\).

\( D \): measure of rate of diffusion process. Called "diffusion coefficient" \([m^2/T]\).

- high \( D \): rapid diffusion.
- low \( D \): slow diffusion.

Three parameters \((t, L, D)\) in two dimensions \((L, T)\) gives one nondimensional number:

\[ P = \frac{D \cdot t}{L^2} \quad \text{or} \quad \frac{L^2}{D \cdot t} \]

\( \downarrow \)

called the Peclet number.
If we want to know how far the paint molecules can diffuse in a time $t$:

\[
\frac{Dt}{L^2} = \text{const}
\]

\[
L = \sqrt{\text{const} \cdot Dt}
\]

\[
L \propto \sqrt{Dt}
\]

"proportional to."

So, if we double the time, the distance will increase by a factor of $\sqrt{2}$.

$\sqrt{Dt}$ : diffusion length scale; characteristic diffusion scale.

$L^2/D$ : diffusion time scale.

We will use these relationships a lot.
Fickian Diffusion:

Diffusion: spreading of substance due to random motion.

(Adolf Fick, 1855)

Diffusion is a fundamental transport process in EFM.

Two primary properties of diffusion:

1. Random in nature (molecular or turbulent)
2. Transport is from high concentration to low concentration.

- Define "flux"

\[ \text{flux} = \text{net rate of flow of (...) through a surface (physical or fictitious)}. \]

Example: mass flux.

\[ \dot{m} = \iint_A \rho \mathbf{u} \cdot \mathbf{n} \, dA \]

- surface

\[ = \iint_A \dot{\rho} \mathbf{u} \cdot \mathbf{n} \, dA \]

- unit normal vector

\[ \dot{m}: \text{total rate of mass flux through } A. \]

\[ \dot{\rho}: \text{mass flux per unit area at a point.} \]

\[ \left[ \frac{M}{L^2 T} \right]. \]

- Particle random motion:

At molecular level, particles interact by Brownian motion.

At turbulent level, superposition of all eddies appears random.
Basic questions:
how far do particles go uninterrupted?
how fast do they travel?

"Random walk"

$\Delta x$: characteristic, random displacement.

$\sim 10^6 \text{nm for gases} \quad \Delta t$: characteristic time scale between collisions.

$\sim 10^7 \text{nm for liquids}$

$\sim \frac{\Delta x}{u_{\text{particle}}}$

$u_{\text{particle}} \sim 0(1000 \text{ m/s})$

No memory effect.

Next step does not depend on previous step.

Consider 1D Motion:

```
\begin{align*}
&\text{Before} \quad &\text{After} \\
&t_0 \quad &t_0 + \Delta t \\
&\uparrow \quad &\uparrow \ \\
&\uparrow \quad &\uparrow \ \\
&\uparrow \quad &\uparrow \ \\
&\uparrow \quad &\uparrow \ \\
&\uparrow \quad &\uparrow \ \\
&&\text{"Painted" molecules.} \\
&\text{"A"} \quad &3 \quad 4 \quad 4 \quad 3
\end{align*}
```
statistically, on average half move left and half move right, otherwise, there would be bias.
Instantaneously: all could move same direction.

* Show diffusion animation.

- Observations:
  - Maximum concentration decreases.
  - Net flux is from high to low concentration.
  - End result will be an equilibrium state of uniform concentration.

- Define "Brownian Motion":
  Apparently random motion of particles due to their deterministic motions and collisions with neighboring particles having random positions and velocities.

**Fick's Law:**

"law" means what we observe in nature. Diffusion is physical transport, not any kind of transformation.