Air-water interface

Now that we know the flux through the interface, we need an equation for how concentrations change inside the water body.

The concentration in the air and water must be the same at the interface. But in concentration units they appear different.

Flux from air into water must equal flux into water.

Diffusion coefficients in air are much higher than in water → the flux into the water body is the rate-limiting step.

Control volume in water:

\[
\frac{dM}{dt} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}
\]

\[
B h \delta x \frac{dc_w}{dt} = B \delta x \left. J_a \right|_{z=0} = B \delta x \ k_a (C_{i,w} - C_w)
\]

\[
\frac{dc_w}{dt} = k_a (C_i - C_w)
\]

units \([1/\text{t}]\)
\(K_e\): gas transfer coefficient.

Oxygen: Aeration (Reaeration)

QUAL2e: a famous EPA water quality model. Uses the following expression, derived by experiment.

\[
K_e \mid_{o_2} = \frac{3.9 \sqrt{\bar{u}}}{H^{3/2}} \\
\bar{u} \text{ in } \frac{m}{s} \\
H \text{ in } m
\]

Beware of units! \(K_e \mid_{o_2}\) in \(\frac{1}{\text{day}}\).

Example: Volatile Chemical in water.

Benzyl Chloride: \(\text{CH}_2\text{Cl}\)

Virtually not present in Atmosphere: \(C_i = 0\). Volatile \(\rightarrow\) readily sublimates into atmosphere.

\[
\begin{align*}
\rightarrow & \quad m \\
\rightarrow & \quad Q, B, h \\
\uparrow \text{assume } m(x=0) &= \frac{1 g}{s}\text{ of } \text{CH}_2\text{Cl} \\
K_e &= 1 \cdot 10^{-4} \text{ s}^{-1} \\
\bar{u} &= 1 \text{ m/s} \\
Q &= 10 \text{ m}^3/\text{s}
\end{align*}
\]

\[
\frac{dC_w}{dt} = -(1 \cdot 10^{-4} \text{ s}^{-1})(C_w)
\]

\[
C(x=0) = C_0 = \frac{m}{Q} = \frac{1 g}{s} \cdot \frac{1 \text{ s}}{10 \text{ m}^3} = 0.1 \frac{\text{mg}}{\ell}
\]
Convert time to space:

\[ x = ut \]
\[ t = \frac{x}{u} \]

Solution:

\[ C_w(x) = C_0 \exp(-K_E t) \]
\[ = 0.1 \frac{mg}{l} \exp(-1.1 \times 10^{-4} \frac{x}{u}) \]

\[ C_w(x) = 0.1 \exp(-0.0001x) \left[ \frac{mg}{l} \right] \]

\[ x \text{ in [m]} \]