Lecture 14: Taylor dispersion.

Review of "Well-mixed" criteria:

Injection at the surface

\[ C_{\text{max}} \text{ only equals } C_{\text{min}} \text{ at } x = \infty \text{ (or } t = \infty). \]

Define "well-mixed" before \( x = \infty \) by an arbitrary level of acceptable error:

\[ \frac{C_{\text{min}}}{C_{\text{max}}} = \beta \text{ near 1.0} ; \text{ say 0.95} \]

If \( \beta = 0.95 \), there is a 5% variation in \( C \) across the cross-section.

We also see that the gradient in \( C \) is proportional to \( \sigma \). As \( \sigma \) increases, \( C \) is closer to well-mixed. We expect there to be a relationship between \( h \) and \( \sigma \):

\[ h = \alpha \sigma \text{ at "well-mixed" condition.} \]

This means cloud has grown to fill region \( x \).
Stagnant flow:
\[ h = \alpha \sqrt{2D t_{w}} \text{ time to become well-mixed.} \]

Uniform channel flow:
\[ t_{w} = \frac{x_{m}}{U} \text{ distance to become well-mixed.} \]

\[ h = \alpha \sqrt{2D \frac{x_{m}}{U}} \]

The coefficient \( \alpha \) depends on the \( \beta \) we pick and on the location where we inject the source \((z_0)\).

Estimating \( D \) from Dye-Study data:

\[ C \]

\[ \sigma_1 = \frac{\sigma_{1L} + \sigma_{1R}}{2} \frac{U}{\bar{U}} \]

\[ \sigma_2 = \frac{\sigma_{2L} + \sigma_{2R}}{2} \frac{U}{\bar{U}} \]

\( \sigma \) is related to \( D \) by:
\[ \frac{\sigma_2^2 - \sigma_1^2}{\Delta t} = 2D \]
Cowaselon Creek Data:

longitudinal Diffusion: \[ \bar{u} = \frac{x}{\Delta t} \]

at Station 2: \[ \sigma_{le} + \sigma_{lr} = 52.5 \text{ min} \]
\[ \sigma_1 = \frac{\sigma_{le} + \sigma_{lr}}{2} \bar{u} = \frac{52.5 \text{ min} \left( \frac{60 \text{ s}}{\text{min}} \right) (0.17 \text{ m/s})}{2} \]
\[ = 268 \text{ m} \]

at Station 3: \[ \sigma_{2l} + \sigma_{2r} = 87.8 \text{ min} \]
\[ \sigma_2 = \frac{\sigma_{2l} + \sigma_{2r}}{2} \bar{u} = \frac{87.8 \text{ min} \left( \frac{60 \text{ s}}{\text{min}} \right) (0.17 \text{ m/s})}{2} \]
\[ = 448 \text{ m} \]

\[ \frac{\sigma_2^2 - \sigma_1^2}{\Delta t} = 2D_x \]
\[ \frac{448^2 - 268^2}{3.97} = 2D_x \]

\[ D_x = 16,231 \text{ m}^2 \cdot \frac{6}{60 \text{ min}} \cdot \frac{1 \text{ s}}{60 \text{ min}} \]
\[ = 4.5 \text{ m}^2 \text{ s}^{-1} \gg D_{b,y} \]

Extra longitudinal diffusion comes from dispersion.