Lecture 25: Viscosity, Bernoulli equation

FLUID MECHANICS

Viscosity: the property of resistance to flow in a fluid.

Consider a stack of copy paper laying on a flat surface. If you give a horizontal push near the top, it will deform but also resist your push.

Think of a fluid as also being composed of layers. When one layer moves relative to another there is a resisting force.

This frictional resistance to a shear force and to flow is called viscosity. It varies by fluid and temperature.

Deformation of a fluid:

- a) Solid.
  - Deforms a finite amount to resist force F

- b) Fluid
  - Continually deforms under force F.

Definition of a fluid: A substance that continually deforms under the action of a shear force.
Newtonian Fluid: A fluid in which the shear stress is directly proportional to the rate of deformation:

\[ \tau = \mu \frac{du}{dy} \]

[\( \frac{F}{A} \)] shear stress [\( \frac{FT}{A} \)] dynamic viscosity coefficient.

Examples: water, refrigerants, and most hydrocarbon fluids (oils, gas, etc).

Non-Newtonian fluids: toothpaste, ketchup, some paints.

No-slip boundary condition: condition that fluids touching a boundary may not have a finite velocity relative to the boundary.

Consider Molecular scale:

\[ \begin{array}{c}
\text{strong forces hold these layer and prevent slipping.} \\
\text{Far from boundary, the fluid moves:}
\end{array} \]

\[ u(y) \]

\[ \text{no-slip b.c.} \]

\[ \mu: \text{intensive quantity, temp. dependent.} \]

Find in Fig. 9-10. Note: \[ \frac{Ns}{m^2} = 2.08 \cdot 10^{-2} \frac{lb \cdot s}{ft^2} \]
Example Problem:

Lubricant is SAE low oil or H2O.

1 m² plate (into page).
Relative velocity: \(0.5 \text{ m/s}\).
\(T = 40^\circ \text{C}\).

Fig. 9-10: \(\mu_{\text{SAE low}} \text{ at } 40^\circ \text{C} = 0.07 \frac{\text{Ns}}{\text{m}^2}\)
\(\mu_{\text{H}_2\text{O}} \text{ at } 40^\circ \text{C} = 0.0065 \frac{\text{Ns}}{\text{m}^2}\)

\[
F_0 = \tau A = \mu \frac{\text{d}u}{\text{d}y} A
\]

\[
= 0.07 \frac{\text{Ns}}{\text{m}^2} \left( \frac{0.5 \text{ m}}{\text{s}} \frac{1}{0.02 \text{ m}} \right) (1 \text{ m}^2)
\]

\[
= 1.75 \text{ N} \text{ for SAE low oil.}
\]

\[
F_w = \tau A = \mu \frac{\text{d}u}{\text{d}y} A
\]

\[
= 0.0065 \left( \frac{0.5}{0.02} \right) (1)
\]

\[
= 0.163 \text{ N}
\]

Oil is more viscous (higher \(\mu\)); thus, friction is higher.

? Why not use water to lubricate your automobile engine?
A viscosity meter (viscometer) is built on the principle of the example:

\[ F = c \tau A = \mu \frac{dw}{dy} A \]

\[ = \mu \omega R \frac{2\pi RL}{d} = \mu \omega R^2 \frac{2\pi L}{d} \]

Torque \( T = FR \)
Measure \( T \); then:

\[ M = \frac{Fd}{\omega R^3 2\pi L} \]

**Bernoulli Equation:** Equation for steady flow in open system.

**Energy Equation:**

\[ \frac{dE_{ex}}{dt} = \dot{Q} - \dot{W} + \dot{m} \left( h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right) \]

For steady flow:

\[ \dot{Q} - \dot{W} = \dot{m} \left( h_2 - h_1 + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \right) \]

\[ q - w = h_2 - h_1 + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \]

Expand Enthalpy term:

\[ q - w = u_2 - u_1 + p_2 v_2^2 - p_1 v_1^2 + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \]
Assume incompressible Flow: \( \nu_1 = \nu_2 \)

\[ q - w = u_2 - u_1 + \nu (P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]

Consider adiabatic flow, no work interaction, isothermal fluid condition:

- \( q = 0 \): Adiabatic.
- \( w = 0 \): No work.
- \( \Delta u = 0 \): Iso-thermal. This means no friction.

Also substitute \( \nu = \frac{1}{g} \):

\[ 0 = (P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]

Rearrange:

\[
\frac{P_2}{g} + \frac{V_2^2}{2} + gZ_2 = \frac{P_1}{g} + \frac{V_1^2}{2} + gZ_1 = \text{Const.}
\]

C Bernoulli Equation: Special form of the Energy Equation.

It can be shown that all our assumptions are met in a frictionless flow only if we follow a streamline.

**Streamline:** a continuous line drawn in the direction of the velocity vector at each point in the flow.

Streamlines.

Stagnation point.
Arrange Bernoulli Equation with units of Pressure:

\[ P + \frac{\rho V^2}{2} + \rho g z = \text{const.} \]

- static pressure
- velocity pressure; elevation pressure.
- hydrostatic pressure; dynamic pressure.

\[ \text{static pressure gauge: reads pressure in pipe.} \]

streamlines. For frictionless pipe flow, these are all the same.

Pitot static tube or Pitot tube:

Let \( z_1 = z_2 \) along stream line:

\[ P_1 + \frac{\rho V_1^2}{2} = P_2 + \frac{\rho V_2^2}{2} \]

\[ \text{stagnation pressure} = \text{Static Pressure} + \text{Dynamic Pressure}. \]

\[ \text{Static Press.} \quad \text{Stagnation Pressure}. \]

Pitot tube: used to measure velocity.

Teamplay: Find stagnation pressure at leading edge of wing.

4-40 MPH

\[ P_{\text{atm}} = 13.7 \text{ psia} \]

\[ T = 60^\circ \text{F} \]
\[ P_{\text{static}} = P_{\text{atm}} = 13.7 \text{ psia}. \]

Density from IGL:

\[ p_v = RT \]

\[ g = \frac{P}{RT} = \frac{13.7 \text{ psia}}{0.3704 \text{ psia ft}^2/\text{lbm R}} \frac{460 + 60}{(460 + 60)} \]

\[ = 0.0711 \frac{\text{lbm}}{\text{ft}^3} \]

\[ P_{\text{static}} + P_{\text{dyn}}. \]

\[ = 13.7 \text{ psia} + \frac{gV^2}{2} \]

\[ = 13.7 \text{ psia} + 0.0711 \frac{\text{lbm}}{\text{ft}^3} \frac{(40 \frac{\text{mi}}{\text{hr}} \cdot 5280 \frac{\text{ft}}{\text{mi}} \cdot 1 \frac{\text{hr}}{3600 \text{s}})^2}{2} \]

\[ = 13.7 \text{ psia} + 122.4 \frac{\text{lbm}}{\text{ft} \cdot \text{s}^2} \]

\[ = 13.7 \frac{\text{lbf}}{\text{in}^2} + 122.4 \frac{\text{lbm}}{12 \text{in} \cdot \text{s}^2} \frac{1 \text{lbf} \cdot \text{s}^2}{32.174 \text{ lbm ft} \cdot 12 \text{in}} \]

\[ = 13.7 \text{ psia} + 0.0264 \text{ psia} \]

\[ P_{\text{stag}} = 13.73 \text{ psia} \]

Arrange Bernoulli Equation with units of distance.

\[ \frac{P}{sg} + \frac{V^2}{2g} + z = \text{const.} \]

\[ \uparrow \quad \uparrow \quad \uparrow \quad \text{elevation head} \]

\[ \text{pressure head} \quad \text{velocity head} \]
Term "head" to express pressure in terms of height appears to originate from reference to the pressure between the head or headwaters of a stream and another point downstream, as in a fountain.

Example: Draining tank. (Neglect friction).

Write Bernoulli Eq at 1:
\[ P_1 + \frac{V_1^2}{2} + gZ_1 = \text{const.} \]
\[ P_{\text{atm}} + \Phi + gZ_1 = \text{const.} \]

Write Bernoulli Eq at 2:
\[ P_2 + \frac{V_2^2}{2} + gZ_2 = \text{const.} \]
\[ P_{\text{atm}} + \frac{V_2^2}{2} + gZ_2 = \text{const.} \]

Are 1 and 2 along streamline? Yes:
\[ P_{\text{atm}} + gZ_1 = P_{\text{atm}} + \frac{V_2^2}{2} + gZ_2 \]
Solve for \( v_2 \):
\[
v_2 = \sqrt{2g(z_1 - z_2)}
\]

Substitute values:
\[
v_2 = \sqrt{2 \left( \frac{32.17 \text{ ft}}{\text{s}^2} \right) \left( 16 \text{ ft} \right)}
\]
\[
= 32.1 \frac{\text{ft}}{\text{sec}}
\]

Flow Rate:
\[
Q = \dot{V} = VA
\]
\[
= 32.1 \frac{\text{ft}}{\text{s}} \cdot \frac{\pi}{4} \left( \frac{4\text{ in}}{12\text{ in/ft}} \right)^2
\]
\[
= 2.8 \frac{\text{ft}^3}{\text{s}}
\]