Lecture 3: Conservation of mass, state postulate, zeroth law, temperature.

**Conservation of Mass:**

"Mass is neither created nor destroyed."

\[ \frac{dM}{dt} = \sum \text{mass inflows} - \sum \text{mass outflows}. \]

\[ \frac{d}{dt} \left( \int \rho \, dV \right) = \sum \left( \int_A \rho V \, dA \right)_{\text{in}} - \sum \left( \int_A \rho V \, dA \right)_{\text{out}} \]

**Simplifications:**

1. Incompressible: \( \rho = \text{const.} \)
   \[ \frac{d}{dt} \left( \int \rho \, dV \right) = \rho \left[ \sum \left( \int_A \rho V \, dA \right)_{\text{in}} - \sum \left( \int_A \rho V \, dA \right)_{\text{out}} \right] \]

2. Uniform velocity over x-sect. \((+ \rho = \text{const.})\):
   \[ \frac{d}{dt} = \rho \sum V_{\text{in}} A_{\text{in}} - \rho \sum V_{\text{out}} A_{\text{out}} \]

3. Steady flow: \( \frac{dV}{dt} = 0 \):
   \[ \rho \sum V_{\text{in}} A_{\text{in}} = \rho \sum V_{\text{out}} A_{\text{out}} \]

4. Ideal gas: \( \rho = \frac{P}{RT} \Rightarrow \frac{PV_1 A_1}{T_1} = \frac{P_2 V_2 A_2}{T_2} \)
A.) Mass flow rate.
Assume uniform, incompressible flow:

\[ \dot{m} = \frac{VA}{v} \]

\[ = \frac{21 \text{ ft/s}}{0.831 \text{ ft}^3/\text{lb}_m} \times 1 \text{ ft}^2 \times \text{lb}_m \]

\[ \dot{m} = 25.3 \text{ lb}_m/\text{s} \]

B.) Exit velocity.
Assume steady flow device:

\[ \dot{m}_{in} = \dot{m}_{out} \]

\[ 25.3 = \frac{V_{out} \times 140 \text{ ft}^2}{175.8 \text{ ft}^3} \]

\[ V_{out} = \frac{25.3 \times 175.8}{140} = 31.7 \text{ ft/s} \]

Goto Slides 8-9.
Example from HW1:

1. A tank and pipe system is given, the initial discharge velocity and the time to empty the tank are to be determined.

2. Assumptions:
   A) Valve is at end of pipe and pipe is initially full of water when tank is 2m deep.
   B) Water pipe is horizontal.
   C) Fluid is incompressible.

3a. Initial velocity:

   \[ v = \sqrt{\frac{2gz}{1.5 + \frac{fL}{D}}} \]

   \[ g = 9.81 \text{ m/s}^2 \]
   \[ z = 2 \text{ m} \]
   \[ f = 0.015 \]
   \[ L = 100 \text{ m} \]
   \[ D = 0.10 \text{ m} \]

4a. Substitution:

   \[ v_o = \sqrt{\frac{2(9.81)(2.00)}{1.5 + 0.015(100)}} = \sqrt{\frac{39.24}{16.5}} \]

   \[ v_o = 1.54 \text{ m/s} \]
3b. Cons. of Mass:

\[
\text{Change in tank storage} = \Sigma \text{in}_m - \Sigma \text{in}_{out}
\]

\[
\frac{dV}{dt} = \rho - \Sigma \rho V_{out} A_{out}
\]

\[
\frac{dV}{dt} = -VA
\]

\[
\pi R^2 \frac{dz}{dt} = -\sqrt{\frac{2gz}{1.5 + \frac{fL}{D}}}
\]

\[
dt = -\frac{R^2}{r^2} \sqrt{\frac{1.5 + \frac{fL}{D}}{2g}} z^{-\frac{1}{2}} dz
\]

4b. \[
\int_0^t dt = \int_0^{-\frac{R^2}{r^2}} \sqrt{\frac{1.5 + \frac{fL}{D}}{2g}} \cdot z^{-\frac{1}{2}} dz
\]

\[
= \frac{-5^2}{0.05^2} \sqrt{\frac{1.5 + 0.015 \cdot 100}{0.1}} \left( z^{\frac{1}{2}} \right)_2^0
\]

\[
= -9170 \left( z^{\frac{1}{2}} \right)_2^0 \cdot \frac{1\text{min}}{60\text{s}} \cdot \frac{1\text{hr}}{60\text{min}}
\]

\[
t = 7.21 \text{ hr}
\]
Equilibrium:

A system is at equilibrium if its properties do not change at any point in the system.

- Thermal equilibrium: temperature does not change with time.
- Mechanical equilibrium: pressure does not change with time.
- Phase equilibrium: mass of each phase does not change with time.
- Chemical equilibrium: molecular structure does not change with time.

No unbalanced potentials (driving forces) in the system.

Individual molecules can change, but there are an equal number going each direction.

State Principle or State Postulate:

"The state of any simple, compressible system is completely given by two independent, intensive properties. Independence: one can stay constant while the other varies.

This only applies at equilibrium.

Process: Change in state of a system from one equilibrium state to another.

Path: Series of states through which a system passes."
Process/Cycle Diagrams:

Plot two intensive properties (i.e. P-v, T-v, P-T)

Alternate path.

Cycle: closed loop → initial and final state is the same.

Process Path

state: each state is independent of the process path to achieve that state.

Constant Property Process:

use prefix "iso" to indicate that a property remains constant along a process path.

- isothermal (temperature)
- isobaric (pressure)
- isochoric or isometric (volume)

Steady-state: constant in time.
Equilibrium: steady in space & time.

How do properties change in time?

Aus: Quasiequilibrium process.

Quasiequilibrium process:

A fictitious, idealized process in which each step along the process path is in equilibrium and in which the state is always defined as in the state postulate.

Approximated in "slowly" varying systems.
Quasiequilibrium Process:

Thought exp.: what if we suddenly remove all the masses.

- small masses slowly removed during expansion of gas.
- departure from equilibrium is always infinitesimally small.

Useful:
- easy to analyze.
- optimal work characteristics achieved by quasiequilibrium process.

State Principle:
State of system defined by \( n + 1 \) intensive properties, where "\( n \)" is the number of relevant quasiequilibrium work modes.

1. is for heat transfer.

\( n \): for instance, volume changes.

Simple, compressible system:
System where volume changes are only relevant quasiequilibrium work mode that can change system properties.

\( \Rightarrow \; n = 1 \); we neglect gravity, magnetism, capillarity, elasticity, external forces, etc.

Other intensive properties are functions of two selected intensive properties:

\[ p_0 = f(p_1, p_2) \] 3 equation of state.

\( \Rightarrow \) not always easy to find.