OCEN 678
Fluid Dynamics for Ocean and Environmental Engineering
Exam 2: Viscous Flows

Date distributed: 10.28.2009
Date due: 10.30.2009 at 1:50 p.m.

Return your solution either in class or in my mail box (8th Floor, CE/TTI) by the date shown above. Please show all your work and follow the rules outlined in the course syllabus.

Instructions:

Please show all your work. Remember that this is an exam. As such, it will be considered cheating if you discuss any part of this exam with anyone other than Dr. Socolofsky. You may reach Dr. Socolofsky by email at socolofs@tamu.edu or by phone at 979-204-0158. You may also stop by his office in CE/TTI 802B. Do not expect to receive confirmation of results or hints on how to solve the problems; only questions for clarification will be answered.

This exam is open book and open notes/homework. You may use the library and any online materials. The only materials that are prohibited are written notes/homework or verbal communications you receive from other students.

*If you use an intermediate result without deriving it, please provide an appropriate citation. Do not skip steps that are integral to the solution of a problem.*

Be sure to answer all parts of all problems.

Certification:

“An Aggie does not lie, cheat, or steal or tolerate those who do.” By my signature below, I certify that the work contained in this exam is my own and that I did not receive help from other students.

Name: ____________________________ Date: ___________________
1 Boundary Layer Thicknesses (30 points)

An approximate solution to the Blasius boundary layer found using the momentum integral is

\[
\frac{u}{U} = 2 \frac{y}{\delta} - \left( \frac{y}{\delta} \right)^2
\]

(1)

where \( u \) is the velocity in the boundary layer as a function of height \( y \), \( U \) is the uniform outer flow velocity, and \( \delta \) is the boundary layer thickness given by

\[
\frac{\delta}{x} = \sqrt{\frac{30}{Re_x}}
\]

(2)

where \( Re_x = Ux/\nu \) and \( \nu \) is the kinematic viscosity of the fluid. From the homework, we know that the displacement thickness \( \delta^* \) and momentum thickness \( \theta \) are

\[
\delta^* = \frac{\delta}{3}
\]

\[\theta = \frac{2\delta}{15}
\]

(3)

Integrate the velocity profile from 0 to \( \delta \) to find the total volume and momentum flux in the boundary layer. Compare these values to the volume and momentum flux for a boundary layer reduced in height by the thicknesses \( \delta^* \) and \( \theta \) and having the uniform velocity \( U \) throughout the reduced boundary layer. Thus, give a physical interpretation to \( \delta^* \) and \( \theta \).

2 Exact Solutions

The flow induced in a deep channel subject to a constant wind forcing can be approximated by the idealized flow problem depicted in Figure 1. The two side walls at \( y = \pm L/2 \) are stationary and impermeable. The top wall at \( z = 0 \) is impermeable and moving at a constant velocity \( U \) in the positive \( x \)-direction (into the page). The flow extends to \( z = \infty \) in the \( z \)-direction and \( x = \pm \infty \) in the \( x \)-direction. Assume there is no pressure gradient in the \( x \)-direction, the fluid is incompressible, and the fluid is stationary except for the motion that is induced by the moving surface.

2.1 Dynamic Pressure (20 points)

State the assumptions necessary to remove the body force and hydrostatic pressure from the governing equations so that we solve only for the dynamic part of the pressure term. Write the Navier-Stokes equations (mass and momentum conservation) under these conditions and define the variables. Is this set of equations an appropriate set to solve the problem stated above. Why or why not?

2.2 Modified Stokes 1st Problem (20 points)

If \( L = \infty \), then this problem reduces to the steady solution to Stokes 1st problem. Show that in this case the governing equations reduce to

\[
\nu \frac{d^2 u}{dz^2} = 0
\]

(4)
subject to the boundary conditions

\[ u(x, y, 0) = U \]
\[ u(x, y, \infty) = U \]  

(5)

Explain the boundary condition at \( z = \infty \). Solve this equation and compare its solution to the unsteady solution to Stokes 1st problem given by

\[ \frac{u}{U} = 1 - \text{erf} \left( \frac{y}{2\sqrt{\nu t}} \right) \]  

(6)

in the limit \( t \to \infty \).

2.3 Simplified Equations (30 points)

Beginning with the Navier-Stokes equations (mass and momentum conservation), derive the governing equation for the exact solution to this flow field for \( L \) positive and finite. Be sure to include all simplifications stated in the problem and those that can be deduced from each of the governing equations and boundary conditions. Clearly state any other assumptions you are making, if applicable.

Summarize your final solution by providing a complete list of assumptions and deductions you used to arrive at your governing equation. Where might you look to find solutions to this problem?