1 *Friction Drag*

The fixed rectangular keel of a sailboat is 1.5 m long (in the flow direction) and 1 m wide and moves in water at a speed of 3 m/s. The water is at 5°C ($\nu = 1.546 \cdot 10^{-6}$).

1. If the surface of the keel is smooth, compute the plate drag coefficient, $C_f$ (note: $Re_{cr} = 5 \cdot 10^5$).

2. Compute the total drag on the keel ($\rho = 1023.5$ kg/m$^3$).

3. Redo parts 1 and 2 if the roughness of the keel is $k = 0.24$ mm (refer to Figure 3.12 in Newman (1977)).

2 *Boundary Layer Thickness*

If the boundary layer on the hood of your car behaves as one on a flat plate, estimate how far from the front edge of the hood the boundary layer becomes turbulent. How thick is the boundary layer at this location?

3 *Atmospheric Boundary Layers*

An atmospheric boundary layer is formed when the wind blows over the earth’s surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants $a$ and $n$ depend on the roughness of the terrain. Typical values are $n = 0.40$ for urban areas, $n = 0.28$ for woodland or suburban areas, and $n = 0.16$ for flat open country.
1. If the velocity is 20 ft/s at the bottom of the sail on your boat \((y = 4 \text{ ft})\), what is the velocity at the top of the mast \((y = 30 \text{ ft})\)?

2. If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

3. Plot the dynamic pressure \(\rho u^2/2\) as a function of elevation for a 30 story building on the A&M campus (each story is 3 m tall) if the wind blows at hurricane strength 165 km/h at the top of the building.

4. Use the velocity profile given above to estimate the shear stress at the ground level using the relationship \(\tau_0 = \mu \partial u / \partial x\). Does this appear to work? Why does your equation give strange results? Can you propose a successful method to estimate the shear stress at the bed from this velocity profile? If so, what method? If not, what information is missing that you would require?

4 Sediments dynamics

Microbial activity in lake sediments often depletes the oxygen as microbes (bacteria) biodegrade organic matter that settles on the lake bottom. Wind action over lakes also causes basin-scale waves (called seiches) which can have periods of a few hours. Because of the wave action, oxygen profiles at the sediment-water interface have a periodic variability. Figure 1 shows two microprofiles of \(O_2\) concentration at the sediment bed taken at different times during the seiching motion. Use these figures to answer the following questions. Clearly mark any values you are reading from the figures.

1. What is the concentration gradient \(\partial C / \partial z\) for each profile at the sediment-water interface in mg/(l·mm) ?

2. Assuming a molecular diffusion coefficient of \(D_{O_2} = 0.11 \cdot 10^{-4} \text{ cm}^2/\text{s}\), what is the diffusive flux of oxygen at the sediment-water interface for each profile in mg/(cm²·s)?

3. If profile 1 is associated with a downward flow of water of 0.1 cm/s and profile 2 is associated with an upward flow of water of 0.1 cm/s, calculate the total mass flux of oxygen per cm² at the sediment-water interface for each profile in mg/s.

4. Why do you think the data have more scatter above the sediment interface than below it? Do you think it is appropriate to use a molecular diffusion coefficient to estimate the diffusive flux?

5. How would the oxygen flux at the sediment-water interface change qualitatively if the \(O_2\) concentration in the water column above the sediments increased to 9 mg/l? What if it decreased to 2 mg/l?
Your consulting firm is contracted to monitor a river reach for the accidental release of contaminants from an upstream construction site. You install two concentration probes which record total dissolved solids in the pristine stream. The measurements from the probes are recorded in Figure 2. Use the data in the figure to answer the following questions, and clearly indicate any values you are reading from the figures.

1. Given that the two stations are 5 km apart, estimate the average flow velocity in the stream in m/s from the data. Assume that the clocks on the two probes are synchronized.

2. What type of source conditions do you expect applied to the release (instantaneous
Figure 2: Total dissolved solids concentration measurements at two stations 5 km apart.

3. Assuming the source was an instantaneous point source, estimate the effective diffusion coefficient for longitudinal spreading from the two concentration profiles.

4. From the diffusion coefficient just obtained, estimate the upstream location where the injection occurred.

5. If the construction site you were monitoring is located between 3 and 4 km upstream of station 1, are the data accurate enough to say with certainty whether the release originated from the construction site? Justify your answer quantitatively.

6. If the river cross-section is uniform with a cross-sectional area of 30 m$^2$, what is your estimate of the total mass injected based on these data?


6 Beach contamination

During a rainstorm a sewer system becomes overloaded, forcing the release of untreated effluent into a river for 1 hour. The river is 5 m deep, 30 m wide and has a mean velocity of 5 cm/s during the storm. The sewer outflow is at the side of the river, and mixes in the vertical direction immediately. From previous dye studies in the river, you know that the lateral effective diffusion coefficient is $D_y = 0.004 \text{ m}^2/\text{s}$ and the longitudinal effective diffusion coefficient is $D_L = 0.15 \text{ m}^2/\text{s}$.

Consider the following questions to evaluate the impact of this release on a public beach located 20 km downstream. Be sure to justify all of your answers quantitatively.

1. Should you use an instantaneous or continuous release model?

2. For mixing in the lateral direction, when will the release first touch the opposite bank? (Hint: Use the standard deviation of the cloud spread to estimate the size of the cloud.)

3. At what time does the peak concentration pass the beach?

4. Since we do not know the amount of effluent released, we must monitor the beach site as long as the potential for elevated contamination exists. Over what time interval should the beach be monitored for contamination? (Hint: relate the expected width of the concentration cloud to the time it takes the cloud to pass by the beach.)

References