1 Laminar boundary layers

Assume that the velocity $u(y)$ in the laminar boundary layer on a flat plate has the profile

$$\frac{u(y)}{U} = \sin \frac{\pi y}{2\delta}$$

(1)

where $U$ is the uniform velocity outside the boundary layer, $\delta$ is the boundary layer thickness and $y$ is between 0 and $\delta$.

1. Plot the velocity profile. Plot $u/U$ versus $y/\delta$ from $y/\delta = 0$ to 1. Note, you do not need to know a value for $U$ or $\delta$ to plot this profile in non-dimensional space. Comment on the qualitative similarity in shape to the Blasius boundary layer velocity profile.

2. Using the von Kármán momentum integral equation, show that

$$\frac{\delta}{x} = \frac{4.795}{\sqrt{Re_x}}, \quad Cf = \frac{0.655}{\sqrt{Re_x}}$$

(2)

where $Re_x = Ux/\nu$.

3. Also obtain expressions for $\delta^*$, $\theta$, and $\tau_0$ as a function of $x$ and $Re_x$.

4. Compare these results to the Blasius solution.
2 Momentum integral

Use the momentum integral and the velocity profile
\[
\frac{u}{U} = a + b\frac{y}{\delta}
\]
(3)
to evaluate the boundary-layer thicknesses \(\delta\), \(\delta^*\), and \(\theta\) and the surface shear stress \(\tau_0\) for flow over a flat plate.

3 Friction drag

The fixed rectangular keel of a sailboat is 1.5 m long (in the flow direction) and 1 m wide and moves in water at a speed of 3 m/s. The water is at 5°C \((\nu = 1.546 \times 10^{-6})\).

1. If the surface of the keel is smooth, compute the plate drag coefficient, \(C_f\) (note: \(Re_{cr} = 5 \times 10^5\)).

2. Compute the total friction drag on the keel \((\rho = 1023.5 \text{ kg/m}^3)\).

3. Redo parts 1 and 2 if the roughness of the keel is \(k = 0.24 \text{ mm}\) (refer to Figure 3.12 in Newman (1977)).

4 Boundary layer thickness

If the boundary layer on the hood of your car behaves as one on a flat plate, estimate how far from the front edge of the hood the boundary layer becomes turbulent when traveling by a school (20 mph) and on the freeway (70 mph). How thick is the boundary layer at this location?

5 Ship models

In Homework #3, you computed the boundary layer thickness for a model and field-scale ship using the Blasius solution. Redo your analysis using the 1/7-power law approximation for turbulent boundary layers as follows:

1. A laboratory scale model of a ship has a length \(L = 2 \text{ m}\) long, width \(W = 0.3 \text{ m}\) wide and moves at a velocity \(U = 1.0 \text{ m/s}\) through water at \(T = 20^\circ\text{C}\). Is the flow in the haul boundary layer turbulent? Estimate the boundary layer thickness \(\delta\) and the total friction drag force using the appropriate formulation.

2. A field-scale ship has length \(L = 200 \text{ m}\), width \(W = 30 \text{ m}\) wide and moves at a velocity of 10 m/s. Is the flow in the haul boundary layer turbulent? Estimate the boundary layer thickness \(\delta\) and the total friction drag force using the appropriate formulation.
Figure 1: Photograph of the *M/V Viking Poseidon* owned by Eidesvik MPSV AS for Statoil serving the North Sea.

3. Compare the non-dimensional boundary layer thickness $\delta/L$ and drag force $F_D/(\rho U^2 W L)$. Comment on the ability of the laboratory model to predict the field-scale ship behavior.

6 Ship bio-fowling

Comment on the validity of the following statement: “Barnacles and similar marine growth are unimportant on the surface of a ship, since the size of these organisms is negligible relative to the size of the ship.” Base your comments on the following example.

The *M/V Viking Poseidon* (refer to Figure 1) is a multi-purpose offshore vessel based in Norway and mostly serving the North Sea. The table given below provides a few specifications. The Fuel Consumption data are for hydraulically smooth conditions.

Estimate each of the following:

1. Using Figure 3.12 from Newman (refer to course handouts), estimate the total drag on the ship when traveling at 15 knots.

2. For the same ship engine power output (same driving force as computed above), estimate the reduced speed of the *Viking Poseidon* when covered with barnacles of a typical scale $k = 2$ mm.

3. Estimate the percent added fuel cost per 1000 NM when the ship is covered with barnacles compared to the hydraulically smooth vessel. Assume a diesel cost of $2.59
per gallon and that the engine power output is that from part 1.

4. What is the range (maximum travel distance without refueling) of the ship under each condition (smooth and with barnacles)?

Based on these estimates, do you think it is worth the effort to keep the vessel free of barnacles?

_Viking Poseidon_ specifications:

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<tr>
<th>Specification</th>
<th>Value</th>
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<tr>
<td>Length</td>
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<tr>
<td>Breadth</td>
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<tr>
<td>Draught</td>
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<td>Fuel capacity</td>
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<td>Fuel density</td>
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<td>Fuel consumption</td>
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