1 Sediments dynamics

Microbial activity in lake sediments often depletes the oxygen as microbes (bacteria) biodegrade organic matter that settles on the lake bottom. Wind action over lakes also causes basin-scale waves (called seiches) which can have periods of a few hours. Because of the wave action, oxygen profiles at the sediment-water interface have a periodic variability. Figure 1 shows two microprofiles of O$_2$ concentration at the sediment bed taken at different times during the seiching motion. Use these figures to answer the following questions. Clearly mark any values you are reading from the figures.

1. What is the concentration gradient $\partial C/\partial z$ for each profile at the sediment-water interface in mg/(l·mm)?

2. Assuming a molecular diffusion coefficient of $D_{O_2} = 0.11 \cdot 10^{-4}$ cm$^2$/s, what is the diffusive flux of oxygen at the sediment-water interface for each profile in mg/(cm$^2$·s)?

3. If profile 1 is associated with a downward flow of water of 0.1 cm/s and profile 2 is associated with an upward flow of water of 0.1 cm/s, calculate the total mass flux of oxygen per cm$^2$ at the sediment-water interface for each profile in mg/s.

4. Why do you think the data have more scatter above the sediment interface than below it? Do you think it is appropriate to use a molecular diffusion coefficient to estimate the diffusive flux at the sediment water interface? Why or why not?

5. How would the oxygen flux at the sediment-water interface change qualitatively if the O$_2$ concentration in the water column above the sediments increased to 9 mg/l? What if it decreased to 2 mg/l?
Figure 1: Oxygen profiles above active sediments in a 35 m deep seiching lake. The sediment water interface is at $z = 0$ mm on the vertical axis. Adapted from Lorke et al. (2003).

2 Diffusion equation

Your consulting firm is contracted to monitor a river reach for the accidental release of contaminants from an upstream construction site. You install two concentration probes which record total dissolved solids in the pristine stream. The measurements from the probes are recorded in Figure 2. Use the data in the figure to answer the following questions, and clearly indicate any values you are reading from the figures.

1. Given that the two stations are 5 km apart, estimate the average flow velocity in the stream in m/s from the data. Assume that the clocks on the two probes are synchronized.

2. What type of source conditions do you expect applied to the release (instantaneous
3. Assuming the source was an instantaneous point source, estimate the effective diffusion coefficient for longitudinal spreading from the two concentration profiles.

4. From the diffusion coefficient just obtained, estimate the upstream location where the injection occurred.

5. If the construction site you were monitoring is located between 3 and 4 km upstream of station 1, are the data accurate enough to say with certainty whether the release originated from the construction site? Justify your answer quantitatively.

6. If the river cross-section is uniform with a cross-sectional area of 30 m², what is your estimate of the total mass injected based on these data?
3 Superposition

If there are two point sources released simultaneously, how do you obtain the concentration field as a function of space and time? You need to prove why your particular method can be applied. If one point source is at \( x = -L \) while the other is at \( x = L \), what is the concentration at \( x = 0 \) (write the equation you would use to solve for \( C \) given \( D, M, A, \) and \( t \))? Plot your result as a function of time with the values of \( D, M \) and \( A \) set as 1.0.

4 Accidental spill in a river

A tanker truck has an accident and spills 100 kg of kerosene into a river. The spill occurs over a span of 3 minutes and can be approximated as uniformly distributed across the lateral cross-section of the river. A fish farm has its water intake 2.5 km downstream of the spill location. Refer to Figure 3.

Figure 3: Schematic of the accidental spill with the important measurement values. \( B \) and \( H \) are the width and depth of the river, \( U \) is the average flow velocity, \( \epsilon \) is the shear velocity, and \( t_0 \) is a non-dimensional number given by:

\[
\epsilon t_0 = 0.145 + \left( \frac{1}{3520} \right) \left( \frac{U}{u_s} \right) \left( \frac{B}{H} \right)^{1.38}
\]

2. What is the length in the downstream direction that the spill occupies due to its
3 minute duration? At what point downstream of the spill do you think it would be reasonable to approximate the spill using an instantaneous point source release?

3. Plot the concentration in the river as a function of downstream distance at $t = 2$ hr after the accident. From the figure, determine the location of the center of mass of the kerosene cloud, the maximum concentration in the river, and the characteristic width of the cloud in the $x$-direction (approximate the cloud using one standard deviation of the concentration distribution).

4. Write the equation for the concentration as a function of time at the inlet to the fish farm. Plot your equation and determine at what time the maximum concentration passes the fish farm.

5. A dye study was conducted in the river at an earlier time and concluded that there is a flow of groundwater into the river along the stretch between the accident and the fish farm. How would this information influence the results reported in the previous steps of this problem?

5 Beach contamination

During a rainstorm a sewer system becomes overloaded, forcing the release of untreated effluent into a river for 1 hour. The river is 5 m deep, 30 m wide and has a mean velocity of 5 cm/s during the storm. The sewer outflow is at the side of the river, and mixes in the vertical direction immediately. From previous dye studies in the river, you know that the lateral effective diffusion coefficient is $D_y = 0.004 \, \text{m}^2/\text{s}$ and the longitudinal effective diffusion coefficient is $D_L = 0.15 \, \text{m}^2/\text{s}$.

Consider the following questions to evaluate the impact of this release on a public beach located 20 km downstream. Be sure to justify all of your answers quantitatively.

1. Should you use an instantaneous or continuous release model?

2. For mixing in the lateral direction, when will the release first touch the opposite bank? (Hint: Use the standard deviation of the cloud spread to estimate the size of the cloud.)

3. At what time does the peak concentration pass the beach?

4. Since we do not know the amount of effluent released, we must monitor the beach site as long as the potential for elevated contamination exists. Over what time interval should the beach be monitored for contamination? (Hint: relate the expected width of the concentration cloud to the time it takes the cloud to pass by the beach.)
Table 1: Drogue position data.

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<th>$y(t_1)$ [km]</th>
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6 Turbulent diffusion in the ocean

Ten surface drogues are released into a coastal region at local coordinates $(x, y) = (0, 0)$. The drogues move passively with the surface currents and are tracked using radio signals. Their locations at the end of $t_1 = 1$ and $t_2 = 20$ days are given in the following table.

1. Estimate the advection velocity and the lateral coefficients of diffusion ($D_x$ and $D_y$) for this coastal region.

2. Using the radio links, the positions of all ten drogues can be collected within ten minutes. Suppose the radio link were to break down and the positions were instead determined through visual observation. Even using a helicopter, it requires nearly four hours to locate all ten drogues. How does this change the accuracy of your data? Can you still consider the measurements to be synoptic?

3. Later, a freight ship is caught in a winter storm off the coast where this drogue study was conducted. High winds and rough seas cause several shipping containers to be washed overboard. One of the containers breaks open, releasing its contents: 29,000 children’s bathtub toys. Estimate how long it will take for the toys to begin to wash up on shore assuming the same transport characteristics as during the drogue study and that the spill occurs 1 km off the coast. (This really happened in the Pacific Ocean, and the trajectory of the bathtub toys, plastic turtles and ducks, were subsequently
used to gain information about the current system.)

References