1 3D Stream Function

In three-dimensional flows, the stream function in spherical coordinates for an axisymmetric flow ($\partial/\partial \omega = 0$) is given by

$$u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \quad u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}$$

(1)

1. What are the corresponding equations for the velocity potential in spherical coordinates?

2. Show that the condition of irrotationality leads to the following condition on $\psi$:

$$r^2 \frac{\partial^2 \psi}{\partial r^2} + \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta} \right) = 0$$

(2)

How does this equation compare to the Laplace equation in spherical coordinates?

3. Explain qualitatively why complex analysis cannot be used in 3D axisymmetric flows to obtain potential flow solutions as it can be for 2D flows.