1 Catalog Problems

Work problems 1.1.8, 1.1.14, and 1.2.9 in the Homework Catalog posted on the Assignments page of the course website. Also work problem 3.5 in the homework catalog for OCEN 677 posted here:

https://ceprofs.civil.tamu.edu/ssocolofsky/OCENx89/assignments.html

2 Vorticity Dynamics

Repeat problem 1.2.6 in the Homework Catalog posted on the Assignments page of the course website, this time with $\rho$ not equal to a constant. What new term(s) appear in the vorticity equation? If a fluid particle starts with zero vorticity, how does this new term affect the vorticity of this fluid particle as we follow the fluid? How is this fundamentally different from the case where $\rho$ is constant?

3 Working with Velocity Fields

The general solution for Couette flow between two infinite, parallel plates with a non-zero, but constant pressure gradient $dp/dx_1$ is given by

$$u_i = \left( \frac{1}{2\mu} \frac{dp}{dx_1} x_2(x_2 - h) + \frac{U x_2}{h} \right), 0, 0$$  \hspace{1cm} (1)

where $U$ is the velocity of the top plate in the $x_1$-direction and $h$ is the distance between the plates in the $x_2$-direction; the bottom plate is stationary and located at $x_2 = 0$. 
1. Substitute this velocity profile into the general equation for the stress tensor for a Newtonian fluid, given by

\[ \sigma_{ij} = -\delta_{ij}p + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

(2)

to find all nine terms of the stress tensor. Write the result as a 3 \times 3 matrix of components.

2. Use our knowledge of the velocity field to obtain a simplified version of the vorticity equation assuming \( u_i = (f(x_2), 0, 0) \) and \( dp/dx_1 \) is constant. Start with the vorticity equation from Problem 1.2.6 in the Homework Catalog, and do not substitute the velocity profile; rather, simplify the governing vorticity equation using these stated assumptions. Solve this simplified vorticity equation to show that the vorticity distribution in the channel must be linear. Verify this result by computing the vorticity directly from the velocity distribution.

3. The energy dissipation as a result of friction for an incompressible Newtonian fluid is given by

\[ \Phi = \frac{\mu}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \]  

(3)

evaluate \( \Phi \) and determine its dimensions for this velocity field. If \( \Phi \) is not exactly zero, what does this mean about the work needed to generate and maintain this steady velocity field?

4. Plot the velocity profile for \( h = 0.25 \) m, \( U = 1 \) m/s and \( dp/dx_1 = 0.1, 0, \) and -0.1 Pa/m. The fluid is water, for which the dynamic viscosity is \( 1 \cdot 10^{-3} \) Ns/m².