Instructions:

This is a 2 hour in-class final exam. You are permitted to consult three sheet of notes, and you may use a hand-held scientific/engineering calculator, pencils, and a ruler. You may also ask me any questions.

Include a sketch and clearly state assumptions and equations used on problems requiring detailed analysis. Failure to do so will result in a lower score. Problems must be worked in the unit system in which they are specified. You may turn in additional sheets as necessary.

Problems that have a “G” in the statement are for graduate students only; problems with a leading “U” are for undergraduates only.

Name: ________________________________

I certify by my signature below that the work I am submitting is my own.

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
1 Wastewater Outfalls

The discharge from a chemical processing plant is planned to be discharged into Corpus Christi Bay. The single discharge port of the diffuser is 10 cm in diameter, will be at 4 m depth, and will flow at 10 l/s. The bay is unstratified with a density of 1020 kg/m$^3$, and the waste stream has a density of 998 kg/m$^3$. The ambient currents are forced by the wind and average 15 cm/s.

a. Calculate the discharge, momentum and buoyancy length scales.

b. (G) given the values of these scales, explain the general physical behavior of the plume (i.e., what will it look like and why?).

c. Calculate the dilution at the end of the near field.

d. What options are available to increase the dilution?

e. (G) the ambient currents vary during the day from peaks around 20 cm/s and lows that are stagnant. What is the range of dilutions (maximum and minimum) that may occur for these currents?

2 Reactive Spill

A traffic accident on a bridge results in the sudden release of 200 kg of Pyrethrin (a botanical insecticide) into the Brazos river (flow rate 30 m$^3$/s, depth 3 m, width 300 m, and slope 0.00001) at one of its banks.

a. How far downstream can the spill be considered well-mixed across the width and the depth?

b. What is the maximum concentration of the insecticide 100 km downstream if it is degraded with a die-off rate of 0.03 d$^{-1}$?

c. (G) If each mole of Pyrethrin degraded consumes 2 moles of oxygen, what would be the oxygen deficit in mg/l at 100 km downstream? The molecular weight of Pyrethrin is 57.0287 g/mol. Do you think that this oxygen anomaly is measureable? Why or why not?

3 Aeration

A wastewater discharge in a river results in an oxygen deficit of 5 mg/l below saturation at a certain water quality station (flow rate 1 m$^3$/s, depth 0.5 m, width 10 m, and slope 0.00001).

a. Using the O’Connor and Dobbins formula ($K = 3.93\sqrt{V}/(d^{3/2})$, where $V$ is in m/s, $d$ is in m, and $k_2$ is in day$^{-1}$), estimate the reaeration constant at the air/water interface.

b. If the measured gas transfer velocity is $k_l = 6 \cdot 10^{-6}$ m/s, how far downstream will half of the oxygen depression be recovered?

c. Solve for the oxygen deficit as a function of distance along the river using $k_l = 6 \cdot 10^{-6}$ m/s.

d. (G) from your solution above, how far downstream does the oxygen deficit become negligible?
A Useful relationships

1 m$^3$ = 1000 l
1 cm$^3$ = 1 ml
1 mg = 1000 µg

The dilution is defined as

$$ S = \frac{C_0}{C} \quad (1) $$

The characteristic length scale of diffusion is

$$ \sigma = \sqrt{2Dt} \quad (2) $$

The Peclet number is defined as

$$ Pe = \frac{D}{uL} \quad (3) $$

Fick’s Law in one dimension is given by

$$ q_x = -D \frac{\partial C}{\partial x} \quad (4) $$

The one-dimensional instantaneous point source solution with advection is

$$ C(x, t) = \frac{M}{A\sqrt{4\piDt}} \exp\left(-\frac{(x - ut)^2}{4Dt}\right) \quad (5) $$

The two-dimensional instantaneous point source solution with advection in the $x$-direction is

$$ C(x, y, t) = \frac{M}{4\pi Ht\sqrt{D_xD_y}} \exp\left(-\frac{(x - ut)^2}{4D_xt} - \frac{y^2}{4D_yt}\right) \quad (6) $$

The steady-state mass flux in a continuous waste stream is

$$ \dot{m} = QC \quad (7) $$

Solution to the first-order rate equation $dC/dt = -kC$ with $C(t) = C_0$

$$ C = C_0 \exp(-kt) \quad (8) $$

For turbulent diffusion

$$ D_{t,z} = 0.067 u_* h \quad (9) $$
$$ D_{t,y} = 0.6 u_* h \quad (10) $$

where $u_* = \sqrt{ghS}$ or $= 0.1\bar{u}$.

Well-mixed criteria for centerline discharges diffusing across a length $L$

$$ L = 2.0\sqrt{2Dt} \quad (11) $$

Well-mixed criteria for edge discharges diffusing across a length $L$

$$ L = 1.1\sqrt{2Dt} \quad (12) $$
Dispersion coefficient according to Fischer et al. (1979)

\[ D_L = 0.011 \frac{u^2B^2}{h u_*} \]  

Dispersion coefficient according to Deng et al. (2001)

\[ D_L = \frac{0.15}{8\epsilon_{t0}} \left( \frac{B}{h} \right)^{5/3} \left( \frac{u}{u_*} \right)^2 u_* h \]  

with

\[ \epsilon_{t0} = 0.145 + \frac{1}{3520} \frac{u}{u_*} \left( \frac{B}{h} \right)^{1.38} \]  

Buoyant plume state variables

\[ Q_0 = u_e \pi \left( \frac{D^2}{2} \right) \]
\[ M_0 = Q_0 u_e \]
\[ B_0 = Q_0 \frac{\Delta \rho}{\rho} g \]

Buoyant plume scales

\[ L_Q = \frac{Q_0}{M_0^{1/2}} \]
\[ L_M = \frac{M_0^{3/4}}{B_0^{1/2}} \]
\[ L_b = \frac{B_0}{u_e^3} \]

Buoyancy dominated near field (no stratification)

\[ SQ_0 = 0.31 \left( \frac{y}{L_b} \right)^{5/3}, \quad \text{when} \quad \frac{y}{L_b} < 5 \]  

Buoyancy dominated far field (no stratification)

\[ SQ_0 = 0.32 \left( \frac{y}{L_b} \right)^2, \quad \text{when} \quad \frac{y}{L_b} \geq 5 \]

Nearfield mixing based on the equation from Huang et al. (1998)

\[ SQ_0 = 0.08 \left( \frac{y}{L_b} \right)^{-1/3} + \frac{0.32}{1 + 0.2(y/L_b)^{-1/2}} \]  

Nearfield mixing with no currents or stratification

\[ S = 0.08 \frac{B_0^{1/3}}{Q_0} y^{5/3} \]

Maximum height of rise (no currents)

\[ z_{max} = 3.98 B_0 \epsilon^{-3/8} \]
where $\epsilon = N^2 = -g/\rho(\partial \rho / \partial z)$.

Maximum height of rise (with currents)

$$z_{\text{max}} = 1.8 \left( \frac{u_a}{\epsilon^{1/2}} \right)^{2/3} \left( \frac{B_0}{u_a^2} \right)^{1/3}$$

(23)

Dilution at the terminal level in stratification (no currents)

$$S = 0.071 \frac{B_0^{1/3}}{Q_0} z_{\text{max}}^{5/3}$$

(24)