Problem Set #2: Fick’s Law and Diffusion

Date distributed: 1/29/09
Date due: 2/5/09 at the beginning of class.

Return your solution either in class or in my mailbox (8th Floor, CE/TTI) by the date shown above. Please show all your work and follow the rules outlined in the course syllabus.

1 Book Questions

Do the following problems from the course text book *Water Quality Engineering in Natural Systems* by David A. Chin:

- Problem 1.2
- Problem 2.2. Also determine whether oxygen solubility increases or decreases with increasing temperature.

2 Concentration Distributions

The concentration profile along a river downstream of a sewage discharge is

\[ C(x) = C_0 \left( 1 - \text{erf} \left( \frac{x}{\sqrt{4Dt}} \right) \right) \]  

(1)

where \( C_0 \) is the initial concentration near the source, \( \text{erf} \) is the error function, \( t \) is the time since the start of release, \( x \) is the distance downstream, and \( D \) is the diffusion coefficient. For this river, \( D = 1 \cdot 10^{-1} \text{ m}^2/\text{s} \) and \( C_0 = 1 \text{ mg/l} \).

- Plot the concentration distribution from \( x = 0 \) to 1500 m at the times \( t = 13.5, 85, \) and 350 hrs. Determine the downstream distance to the points \( x_i \) were \( C = 0.5 \) and 0.37 mg/l for each time.

- Calculate the characteristic length scale \( L = \sqrt{Dt} \) at the times \( t = 13.5, 85, \) and 350 hrs. Compare the results to the locations \( x_i \) found in the previous step where the concentration equals 0.5 and 0.37 mg/l. Can you formulate an expression of the form \( x_i = f(L) \) for \( C = 0.5 \) and 0.37 mg/l? (note: you will obtain two relationships, one for each concentration.)
• From your plots in the previous step, estimate the diffusive flux at the point \( x = 500 \text{ m} \) in \( \mu g/cm^2s \) for \( t = 13.5, 85, \) and 350 hrs. When is the diffusive flux greatest?

• Plot the concentration variation at the point \( x = 500 \text{ m} \) from \( t = 0 \) to 500 hrs. When does the concentration at this point first reach 0.5 mg/l.

3 Integral Version of the Diffusion Equation

Consider a continuously stirred tank reactor (CSTR) with initial concentration \( C_0 = 0 \) having a volume of \( V = 1 \text{ m}^3 \). Phosphate is added to the reactor through an inflow having \( C_{in} = 1 \text{ mg/l} \) and \( Q_{in} = 0.05 \text{ m}^3/s \). The outflow from the reactor is also \( Q_{out} = 0.05 \text{ m}^3/s \) so that the volume of the reactor remains constant.

• Solve for the concentration \( C(t) \) in the CSTR over time and plot the result.

• What is the steady state concentration in the reactor?

• If you want the steady state concentration in the reactor to increase by a factor of 2, what options do you have to achieve this goal?

4 (G) Fick’s Law

Verify your estimates of the diffusive flux in Problem 2 at \( x = 500 \text{ m} \) using Equation (1) and Fick’s Law analytically.

5 (G) Tank Reactors

Solve Problem 3 again, this time assuming the outflow is twice the inflow. Plot the solution for \( C(t) \) and determine when the tank will run dry. Is there a steady state solution for this case?