1 Two-dimensional Instantaneous Point-Source Solution

A boating accident results in a spill of 10 kg of MTBE (a gasoline additive) which mixes rapidly over the depth in a shallow lake and dissolves into the local ambient water. Analyze the mixing of this spill for a lake having a depth of 4 m, a weak current toward the east ($x$-direction) at 5 cm/s, and mixing coefficients $D_x = 0.5 \text{ m}^2/\text{s}$ and $D_y = 0.1 \text{ m}^2/\text{s}$.

- Plot the concentration distribution along the $x$- and $y$-axes, where the spill location is at the point $(0,0)$, for $t = 1$, 3, and 10 days following the spill.
- Plot the concentration at a drinking-water intake 600 m south and 10 m west of the spill site over time.
- When does the maximum concentration occur at the drinking water intake?
- What is the Peclet number for mixing in the $x$-direction? Is this flow diffusion or advection dominant?
- A fish spawning bed is located 1 km east and 350 m south of the spill site. Over what period of time should the site be monitored for MTBE contamination as a result of this spill?

For these problems, you may ignore the lateral boundaries of the lake which are far away from the spill site.

2 Integral Version of the Diffusion Equation

The same lake as in the previous problem can be approximated over large times as a 4 m deep, well-mixed container 15 km by 15 km in area. A river discharges into the lake at a rate of 1 m$^3$/s,
and the net outflow from the lake is also 1 m³/s. A paper mill on the upstream river plans to begin discharge of a mill by-product at a rate of 1 l/s of 500 mg/l solution. Assume that the mill by-product does not undergo any chemical or biological transformations in the lake and that the wind keeps the lake well-mixed.

- Derive an equation for the concentration of mill by-product in the lake over time as a function of the given variables.
- Plot the concentration in the lake over time as the lake progresses from clean conditions to the steady-state by-product concentration.
- What is the steady-state concentration of mill by-product in the lake and when does the lake concentration get “close” to this value?
- When does the concentration in the lake first reach 50% of the steady-state concentration?

3 Undergraduate Student Problems


4 Graduate Student Problems

Solve problems 2.6 and 2.13 in the Socolofsky and Jirka online textbook.