Lec. 21: Isentropic efficiencies, air standard cycle, Carnot cycle, Otto cycle.

Review of Lec. 19: (Lec. 20 was optional review).

Relationships for ideal gases:

Exact analysis:

\[ s_2 - s_1 = \int_{T_i}^{T_2} c_p(T) \frac{dT}{T} - R \ln \left( \frac{P_2}{P_1} \right) \]

\[ s_2 - s_1 = \int_{T_i}^{T_2} c_v(T) \frac{dT}{T} + R \ln \left( \frac{v_2}{v_1} \right) \]

Both equations apply to any situation and come from the Tds relationships.

For isentropic processes: (with \( c_p = \bar{c}_p \), \( c_v = \bar{c}_v \))

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^\frac{k-1}{k} \quad \text{where} \quad k = \frac{c_p}{c_p - R} \]

\[ P_2 v_2^k = P_1 v_1^k \quad \text{const.} \]

For variable specific heats:

\[ s_2 - s_1 = s^0(T_2) - s^0(T_1) - R \ln \left( \frac{P_2}{P_1} \right) \]

Tabulated

**Note:** in table A-17:

\[ P_r = \exp \left( \frac{s^0(T)}{R} \right) \quad \text{and} \quad \nu_r = \frac{T}{P_r} \]

If you need an actual \( P \) or \( \nu \), use \( P \nu = RT \).
Isentropic Efficiencies:

Actual processes with $S_{gen} > 0$ have less heat available to do work than reversible processes where $S_{gen} = 0$. Also, adiabatic processes have no contribution in the $\frac{\delta Q}{T}$ term:

$$\Delta S = \int \frac{\delta Q}{T} + S_{gen}$$

So, isentropic processes are more efficient than real processes that have irreversibilities. Isentropic processes are, therefore, an ideal to which we can compare other processes.

Compressors:

We add work and they compress gases. We want to add the minimum amount of work.

- $W_s =$ work required in isentropic process.
- $W_a =$ actual work required for process.

Efficiency Measure: (depends on device)

$$\eta_c = \frac{W_s}{W_a} \leq 1 \quad \text{and} \quad \geq 0.$$  

Let's analyze a compressor to compute the $W$s:

\[ T_2, h_2, P_2 \quad \text{inlet} \]

\[ W \quad \text{work} \]

\[ \dot{W} = m(\Delta h + \Delta pV + \Delta pE) \]

\[ Q = 0 \]

\[ W_{in} = m(\Delta h) \]

\[ = m(h_2 - h_1) \]

\[ \dot{w}_{in} = (h_2 - h_1). \]
For isentropic compressor:
\[ W_s = \dot{m}(h_s - h_1) \]
\[ \uparrow \text{ output enthalpy in isentropic process.} \]

For actual compressor:
\[ W_a = \dot{m}(h_a - h_1) \]
\[ \uparrow \text{ output enthalpy in actual process.} \]

I can freely choose \( h_1 \) to be the same as the isentropic case.

Combine:
\[ \eta_c = \frac{h_s - h_1}{h_a - h_1} \]

For ideal gas with constant specific heats:
\[ \eta_c = \frac{h_s - h_1}{h_a - h_1} = \frac{c_p(T_s - T_1)}{c_p(T_a - T_1)} = \frac{(T_s - T_1)}{(T_a - T_1)} \]

(Real gases also have dependence on \( \Delta P \)).
**Turbines:**

We are interested in work output and actual process generates less work than isentropic process.

\[ \eta_T = \frac{W_a}{W_s} = \frac{h_1 - h_s}{h_2 - h_a} \quad ; \quad 0 \leq \eta_T \leq 1. \]

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**Nozzle:**

We are interested in high velocity output:

\[ \eta_N = \frac{k_e a}{k_e s} = \frac{V_{2a}^2}{V_{2s}^2}. \]

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**Problem 7-89: Teamplay**

**Compressor:**

\[ T_2 = ? \quad 1 \text{MPa} \]

\[ \eta_c = 0.80 \]

\[ n, \text{ sat. vapor}, 120 \text{ kPa} \]

\[ V = \frac{0.3 \text{ m}^3}{\text{min}} \]

---

**Diagram:**

- T-S diagram
- Pressure and temperature values
- Mass flow rate
\[ \eta_c = \frac{h_s - h_i}{h_a - h_i} = 0.8 \]

Find \( h_s \), then solve for \( h_a \) using above.

Find \( h_s \):

\[ s_1 = s_g @ 120 \text{kPa} = 0.9354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \quad h_1 = h_g @ 120 \text{kPa} = 233.86 \frac{\text{kJ}}{\text{kg}} \]

\[ s_{2s} = s_1 = 0.9354 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \]

\[ p_{2s} = 1 \text{ MPa} \]

Superheated vapor:

\[
\begin{array}{ccc}
T & h & s \\
268.68 & 40 & 0.9066 \\
280.19 & 50 & 0.9428
\end{array}
\]

\( \Rightarrow \) interpolate \( T_{2s} = 47.96 \)

\( h_{52} = 277.837 \frac{\text{kJ}}{\text{kg}} \)

Solve for \( h_a \):

\[ \eta_c = \frac{h_{2s} - h_i}{h_a - h_i} = 0.8 \]

\[ = \frac{277.837 - 233.86}{h_{2a} - 233.86} = 0.8 \]

\[ h_{2a} = 288.83 \frac{\text{kJ}}{\text{kg}} \]

Superheated vapor:

\[
\begin{array}{cc}
T & h \\
50 & 280.19 \\
60 & 291.36
\end{array}
\]

\[ T_{2a} = 57.7 \degree \text{C} \]
Required Work input:

\[
\dot{Q} - \dot{w} = m (h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g \frac{\Delta z}{g})
\]

\[
\dot{w} = m (h_2 - h_1)
\]

Find \( m \):

\[
m = \frac{\dot{V}}{V_1} = \frac{0.3 \frac{m^3}{min} \cdot 1 \text{ min}}{60 \text{ s}} = \frac{0.005 \frac{m^3}{s}}{0.1614 \frac{m^3}{kg}}
\]

\[
= 0.03098 \frac{kg}{s}
\]

Plug in:

\[
\dot{w} = 0.03098 \left( h_{2a} - h_{g@120 \text{kPa}} \right)
\]

\[
= 0.03098 \left( 288.83 - 233.86 \right) \frac{\text{kJ}}{\text{kg}}
\]

\[
= 1.703 \frac{\text{kJ}}{\text{s}}
\]

\[
W = -1.703 \text{ kW}
\]

\[
W = 1.703 \text{ kW input}
\]
Carnot Cycle:
The most efficient cycle (because it's reversible) that can be executed between a heat source and heat sink:

\[ \eta = 1 - \frac{T_L}{T_H} \]

Drawback is that isothermal heat transfer is difficult to obtain in the real world; it requires large heat exchangers and a lot of time.

Gas Cycles:
Alternative cycles that more approximate reality. Cannot cycle efficiency will be used as a standard of comparison.

Assumptions of the Air Standard Cycle:
1. Working fluid is air.
2. Air is ideal gas.
3. Combustion process (explosion) is replaced by heat addition process.
4. Heat rejection is used to restore the fluid to its original state and complete the cycle.
5. All processes are internally reversible.
6. Constant or variable specific heats can be used.

Engineering examples:
- Internal combustion engine
  - Otto and Diesel cycle.
- Gas turbines
  - Brayton cycle.
- Refrigeration
  - Reversed Brayton cycle.
Nomenclature: Show slides ppt 21, 17 & 18.

\( \text{d} \): Bore diameter \([L]\)
\( \text{s} \): Stroke length \([L]\)
\( \Delta V \): Displacement volume \([L^3]\) = \(s \frac{\pi d^2}{4}\)
\( cV \): Clearance volume
\( r \): Compression ratio

\[
 r = \frac{\Delta V + cV}{cV} = \frac{V_{BOC}}{V_{TDC}}
\]

Piston moves up and down.
Values open and close to let working fluid move in and out of cylinder.

Mean Effective Pressure:

Recall boundary work: \(W_b = \int P \Delta V\)
If we assume \(P = \text{const}\): \(W_b = P \Delta V\)

In the actual combustion cycle \(P \neq \text{const.}\), and a net amount of work is done. We can define a fictitious effective pressure as:

\[
\text{MEP} = \bar{P} = \frac{W_{\text{net}}}{\Delta V} = \frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}}
\]

Show slides for Real Otto Cycle. (ppt 21, 22-25).

1-2: Adiabatic compression.
2-3: Const. \(V\) heat addition.
3-4: Adiabatic expansion.
4-1: Const. \(V\) heat rejection.
Otto Cycle Efficiency:

\[ \eta = \frac{W_{\text{net}}}{q_{\text{in}}} \]

Heat addition is at const. \( T \) from state 2 to 3. During this process we have a closed system.

\[ Q - W = (\Delta U_{\text{sys}} + \Delta P_{\text{ext}} + \Delta P_{\text{int}}) M_{\text{sys}} \]

0: piston is stationary for this part of cycle.

\[ \frac{Q}{M_{\text{sys}}} = q_{\text{in}} = \Delta U_{\text{sys}} = u_3 - u_2 \]

Net work from cycle energy balance:

\[ Q - W_{\text{net}} = 0 \quad \text{for a whole cycle.} \]

\[ W_{\text{NET}} = q_{\text{in}} - q_{\text{out}} \]

\[ W_{\text{NET}} = q_{\text{in}} - q_{\text{out}} \]

Heat loss is from state 4-1 at const. volume.

\[ Q - W = \Delta U_{\text{sys}} M_{\text{sys}} \]

\[ -q_{\text{out}} = \Delta U_{\text{sys}} = u_1 - u_4 \]

\[ q_{\text{out}} = u_4 - u_1 \]

Then \( W_{\text{net}} \) must be:

\[ W_{\text{net}} = (u_3 - u_2) - (u_4 - u_1) \]

Substitute into efficiency:

\[ \eta = 1 - \frac{(u_4 - u_1)}{(u_3 - u_2)} \]
Problem 8-36: 

Air Standard Otto cycle.

\[ r = 9.5 \]

1. \( P = 100 \text{ kPa}, \ T = 17^\circ \text{C}, \ \frac{V}{V_0} = 600 \text{ cm}^3 \)

2. \( T = 800 \text{ K} \).

Assume Cold air cycle.

Find: \( T_{\text{max}}, \ P_{\text{max}}, \ q_{\text{in}}, \ \eta, \ \text{MEP} \).

Find mass:

\[
m = \frac{PV}{RT} = \frac{(100 \text{ kPa})(600 \text{ cm}^3)(\frac{1 \text{ m}^3}{100 \text{ cm}^3})}{(0.2870 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(17^\circ \text{C} + 273 \text{K})}
\]

\[ = 0.0007209 \text{ kg} \]

Find \( T_2 \):

\[
T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = (273 + 17) \left( \frac{9.5}{0.5} \right)^{1.4 - 1}
\]

\[ T_2 = 713.65 \text{ K} \]

Find \( T_3 \):

\[
T_3 = \frac{T_4}{\left( \frac{V_4}{V_3} \right)^{k-1}} = (800) \left( \frac{9.5}{0.5} \right)^{1.4 - 1}
\]

\[ T_3 = 1968.7 \text{ K} \]

\( T_{\text{max}} = 1970 \text{ K} \)
Find $P_{\text{max}}$:

$$V_3 = \frac{\pi r^2}{r} = 63.16 \text{ cm}^3$$

$P_{\text{max}} @ T_{\text{max}} = 1970 \text{ K}$

$$P_{\text{max}} = \frac{MRT_{\text{max}}}{V_{\text{min}}}$$

$$= \left(7.209 \cdot 10^{-4} \text{ kg}\right) \left(0.2870 \text{ kJ/kg.K}\right) \left(1970 \text{ K}\right)$$

$$\frac{(63.16 \text{ cm}^3)}{(100^3 \text{ cm}^3)}$$

$$P_{\text{max}} = 6.45 \text{ MPa}$$

Find $Q_{\text{in}}$:

$$Q_{\text{in}} = MC_v(T_3 - T_2)$$

$$= \left(7.209 \cdot 10^{-4} \text{ kg}\right) \left(0.718 \text{ kJ/kg.K}\right) \left(1970 - 713.65\right)$$

$$Q_{\text{in}} = 0.650 \text{ kJ}$$

Find $\eta$:

$$\eta = 1 - r^{k-1}$$

$$= 1 - 9.5^{(1.4-1)}$$

$$\eta = 0.594$$

Find $\text{MEP}$

$$\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = 0.594 = \frac{W_{\text{NET}}}{0.650 \text{ kJ}}$$

$$W_{\text{NET}} = 0.386 \text{ kJ}$$
\[ W_{\text{net}} = \bar{P} (V_2 - V_1) \]

\[ \bar{P} = \frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}} = \frac{0.386 \text{ kJ}}{(600 - 63.16) \text{ cm}^3 \cdot \frac{1 \text{ m}^3}{10^3 \text{ cm}^3}} \]

\[ \bar{P} = 719 \text{ kPa} \]
Cold air standard cycle: approximation to get a "first-cut" estimate of a cycle's efficiency.

→ Assume $c_v$, $c_p$, and $k$ are constant and equal to their values at 70°F (20°C).

Look at Efficiency:

$$\eta = 1 - \frac{u_4 - u_1}{u_3 - u_2} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Note that cycle steps 1→2 and 3→4 are isentropic. Use isentropic ideal gas relationships

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1}$$

$$\frac{T_4}{T_3} = \left(\frac{V_4}{V_3}\right)^{k-1} = \frac{1}{r^{k-1}}$$

$$\Rightarrow \frac{T_4}{T_3} = \frac{T_1}{T_2}$$

Substituting, we get:

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{k-1}}$$

Note that $T_1$ and $T_2$ depend on the design, not the TER; therefore, this is not a Carnot efficiency.
Sample Problem:

Air-Standard Otto Cycle.

1) 95 kPa, 22°C, \( \dot{V} = 5600 \text{ cc} \\
   r = 9 \\
   q_{in Msys} = Q_{in} = 8.6 \text{ kJ}.

Find:

A.) Temp. and pressure at 3.
B.) \( \eta \) for cycle.

Use cold air assumptions.

Solution:

Mass of air in system: IGL!

\[
M_{sys} = \frac{PV}{RT} = \frac{(95 \text{ kPa}) (5600 \text{ cm}^3) (\frac{1 \text{ m}^3}{1000 \text{ cm}^3})}{(0.2870 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(273.15 \text{ K} + 22 \text{ C})} \\
= 6.28 \cdot 10^{-3} \text{ kg}.
\]

Find \( T_2 \):

\[
\frac{T_2}{T_1} = \left( \frac{\dot{V}_1}{\dot{V}_2} \right)^{k-1} = r^{k-1} = 9^{(1.4-1)} = 2.41
\]

\[ \Rightarrow T_2 = 710.79 \text{ K} \]

Find \( T_3 \): \( 2 \rightarrow 3 \) is heat addition process.

\[
Q - W = (\Delta u + \Delta ke + \Delta pe) M_{sys}
\]

\[
Q_{in} = M_{CV} (T_3 - T_2)
\]

\[
T_3 = \frac{Q_{in}}{M_{CV}} + T_2 = \frac{8.6 \text{ kJ}}{(6.28 \cdot 10^{-3} \text{ kg}) (0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})} + 710.79 \text{ K}
\]

\[
T_3 = 2618 \text{ K}
\]
Thermal Efficiency:

\[ \eta = 1 - \frac{1}{\pi^*-1} = 1 - \tilde{\eta}^{(1.4-1)} \]

\( \eta = 0.585 \)

Show diesel cycle slides. ppt 22, 42-44.